NORTH MAHARASHTRA UNIVERSITY,

JALGAON

Question Bank

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Class : S.Y. B. Sc. Subject : Mathematics

Paper : MTH – 212 (B) (Computational Algebra)

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Question Bank

Paper : MTH – 212 (B)

Computational Algebra

Unit – I

1 : Questions of 2 marks

1) Define reflexive relation and irreflexive relation.
2) Define symmetric and antisymmetric relation.
3) Define transitive closure and symmetric closure of a relation R on a set A.
4) Define closure and symmetric closure of a relation R on a set A.
5) Define reflexive closure of a relation R on a set A. Explain by an example.
6) Define rechability relation R^* and a relation R^∞, where R is a relation on a set A.
7) Define a partition of a set. List all partitions of a set A = {1, 2, 3}.
8) Define Boolean product and Boolean addition of two Boolean matrices.
9) Let A = {1, 2, 3, 4} and R = {(1 , 1) , (1 , 2) , (2 , 3) , (3 , 1) , (4 , 3) , (3 , 2)}. Find R(1) , R(2) , R(X) if X = {3, 4}.
10) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Compute $A \vee B$ and $A \wedge B$.

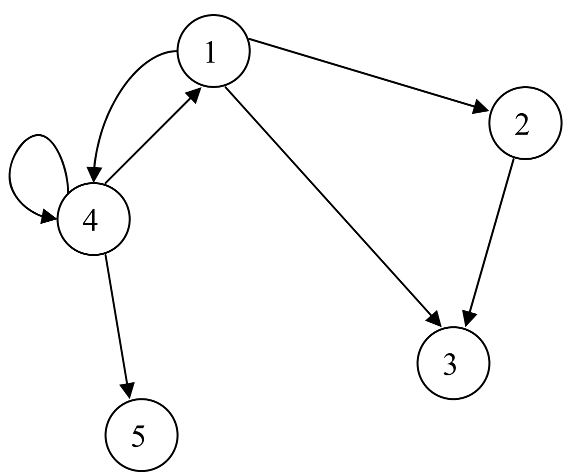
11) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$. Compute $A \odot B$.

12) Let A = {a, b, c, d, e}and R be a relation on A and matrix of relation R is $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$. Find R and its diagraph.

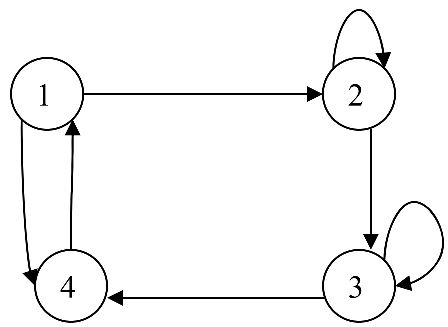
13) If A = {1, 2, 3, 4, 5, 6, 7}and R = {(1 , 2) , (1 , 4) , (2 , 3) , (2 , 5) , (3 , 6) , (4 , 7)}then compute the restriction of R to B = {1, 2, 4, 5}.

14) Let A = {a, b, c, d} and R be the relation on A that has matrix of relation is $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. Construct its diagraph. Also find indegree and outdegree for each vertex.

15) Find the relation and its matrix whose diagraph is given below :



16) For the following diagraph list the indegree and out degree of each vertex. Also write the corresponding relation :



2 : Multiple choice Questions of 1 marks

1) Let A = {1, 2, 3, 4} , B = {1, 4, 6, 8, 9} and R be a relation from A to B defined by aRb $\Leftrightarrow$ $b = a^2$. Then dom(R) = - - - -

a) {1, 2, 3, 4} b) {1, 2, 3}

c) {1, 4, 9} d) {1, 4, 9, 16}

2) Let A = {1, 2, 3, 4} , B = {1, 4, 6, 8, 9} and R be a relation from A to B defined by aRb $\Leftrightarrow$ $b = a^2$. Then Ran(R) = - - - -

a) {1, 2, 3, 4} b) {1, 2, 3}

c) {1, 4, 9} d) {1, 4, 9, 16}

3) Let A = {1, 2, 3, 4, 6, 9, 12} and R be a relation on A defined by aRb $\Leftrightarrow$ a is a multiple of b. Then R-relative set of 6 is - - - -

a) {1, 2, 3, 6} b) {6, 12}

c) {1, 2, 3} d) {12}

4) A relation R on a set A is reflexive if and only if - - - -

a) all diagonal entries of M_R are 1 and non diagonal entries of M_R are 0

b) all diagonal entries of M_R are 1

c) all diagonal entries of M_R are 0

d) all diagonal entries of M_R are 0 and non diagonal entries of M_R are 1

5) A relation R on a set A is irreflexive if and only if - - - -

a) all diagonal entries of M_R are 1 and non diagonal entries of M_R are 0

b) all diagonal entries of M_R are 1

c) all diagonal entries of M_R are 0

d) all diagonal entries of M_R are 0 and non diagonal entries of M_R are 1

6) Let R be a relation on a set A. Then M_{R^2} = - - - -

a) $M_R \oplus M_R$ b) $M_R \vee M_R$ c) $M_R \wedge M_R$ d) $M_R \odot M_R$

7) Symmetric closure of a relation R on a set A is - - - -

a) $\overline{R}$ b) R^{-1} c) $R \cup R^{-1}$ d) $R \cap R^{-1}$.

8) Let A = {1, 2, 3, 4}. Which of the following is a partition of A?

a) {{1,2} , {3}} b) {{1,2} , {3,4}}

c) {{1,2,3} , {2,3,4}} d) {{1,2} , {2,3} , {1,2} , {2,3}}

3 : Questions of 4 marks

1) If R and S are equivalence relations on a set A then show that the smallest equivalence relation containing R and S is $(R \cup S)^{\infty}$.

2) If R is a relation on A = {a_1, a_2, - - - , a_n} then show that $M_{R^2} = M_R \odot M_R$.

3) Let R be a relation on a set A. Prove that R^{∞} is a transitive closure of R.

4) Let A be a set with n elements and R be a relation on A. Prove that $R^{\infty} = R \cup R^2 \cup - - - \cup R^n$.

5) Explain the method of finding partitions A/R, where R is an equivalence relation on a finite set A. Let A = {1, 2, 3, 4} and R = {(1 , 1) , (1 , 2) , (2 , 1) , (2 , 2) , (3 , 4) , (4 , 3) , (3 , 3) , (4 , 4)} be an equivalence relation on A. Find A/R.

6) Let P be a partition of a set A. Define a relation R on A by "aRb if and only if a and b belong to same set in P". Prove that R is an equivalence relation on A.

7) Explain Warshall's algoritham. Using Warshall's algoritham find the transitive closure of a relation R whose matrix is $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

8) Using Warshall's algoritham find the transitive closure of a relation R whose matrix is $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

9) Using Warshall's algoritham find the transitive closure of a relation R whose matrix is $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

10) Compute W_1, W_2, W_3 as in Warshall's algoritham for the relation R on a set A = {1, 2, 3, 4, 5}and matrix of R is $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} = W_0$.

11) Let A = {1, 2, 3} and R = {(1 , 1) , (1 , 2) , (2 , 3) , (1 , 3) , (3 , 1) , (3 , 2)}. Find the matrix $M_{R^{\infty}}$ using the formula $M_{R^{\infty}} = M_R \vee (M_R)^2 \vee (M_R)^3$.

12) Let A = {a, b, c} and R = {(a , a) , (b , b) , (b , c) , (c , b) , (c , c)}. Find the matrix $M_{R^{\infty}}$ using the formula $M_{R^{\infty}} = M_R \vee (M_R)^2 \vee (M_R)^3$.

13) Let A = {1, 2, 3} and B = {a, b, c, d, e, f} and R = {(1 , a) , (1 , c) , (2 , d) , (2 , e) , (2 , f) , (3 , b)}. Let X = {1, 2} , Y = {2, 3}. Show that $R(X \cup Y) = R(X) \cup R(Y)$ and $R(X \cap Y) = R(X) \cap R(Y)$.

14) Let A = {1, 2, 3, 4, 5} and R = {(1 , 1) , (1 , 2) , (2 , 3) , (3 , 5) , (3 , 4) , (4 , 5)}. Compute R^2 , R^{∞} and draw diagraph for R^2.

15) Let A = {x, y, z, w, t} and R = {(x , y) , (x , w) , (y , t) , (z , x) , (z , t) , (t , w)}. Compute R^2 , R^{∞} and draw diagraph for R^2.

16) Let A = {1, 2, 3, 4, 5, 6, 7} and R = {(1 , 2) , (1 , 4) , (2 , 3) , (2 , 5) , (3 , 6) , (4 , 7)} be a relation on A. Find i) R-relative set of 4 ii) R-relative set of 2 iii) restriction of R to B, where B = {2, 3, 4, 5}.

17) Determine the partitions A/R for the following equivalence relations on A

i) A = {1, 2, 3, 4} and R = {(1 , 1) , (1 , 2) , (2 , 1) , (2 , 2) , (1 , 3) , (3 , 1) , (3 , 3) , (4 , 1) , (4 , 4)}.

ii) S = {1, 2, 3, 4} and A = S × S and R be a relation on A defined by $(a , b)R(c , d) \Leftrightarrow ad = bc$.

18) Let A = {1, 2, 3, 4} and R be a relation on A whose matrix is $M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$. Find the reflexive closure of R and symmetric closure of R.

19)Let A = {1, 2, 3, 4}and R be a relation on A whose matrix is M_R = $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$. Find the reflexive closure of R and symmetric closure of R.

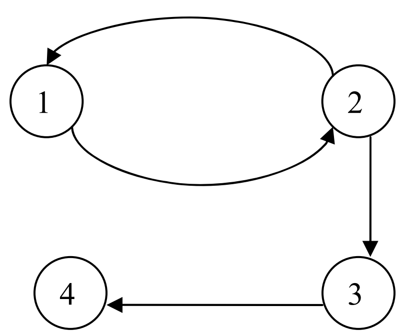
20)Let R, S be relations from A = {1, 2, 3} to B = {1, 2, 3, 4} whose matrices are $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ and $M_S = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$. Find

i) $M_{\overline{R}}$ ii) $M_{\overline{S}}$ iii) $M_{R \cup S}$

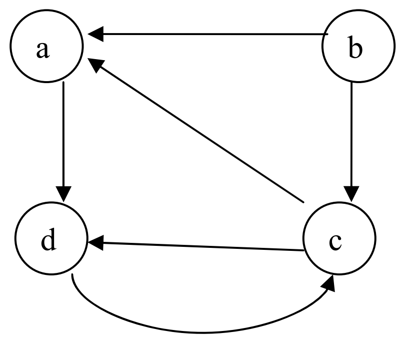
21) Let R, S be relations from A = {1, 2, 3, 4} to B = {1, 2, 3} whose matrices are $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ and $M_S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find

i) $M_{R^{-1}}$ ii) $M_{S^{-1}}$ iii) $M_{(R \cup S)^{-1}}$.

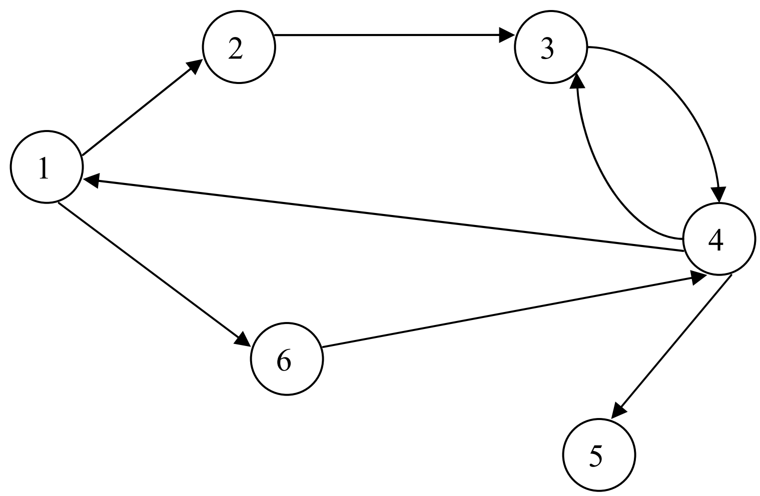
22) Using Warshall's algoritham , find the transitive closure of relation R on a set A = {1, 2, 3, 4} given by diagraph :



23) Using Warshall's algoritham , find the transitive closure of relation R on a set A = {a, b, c, d} given by diagraph :



24) Let R be a relation whose diagraph is given below :

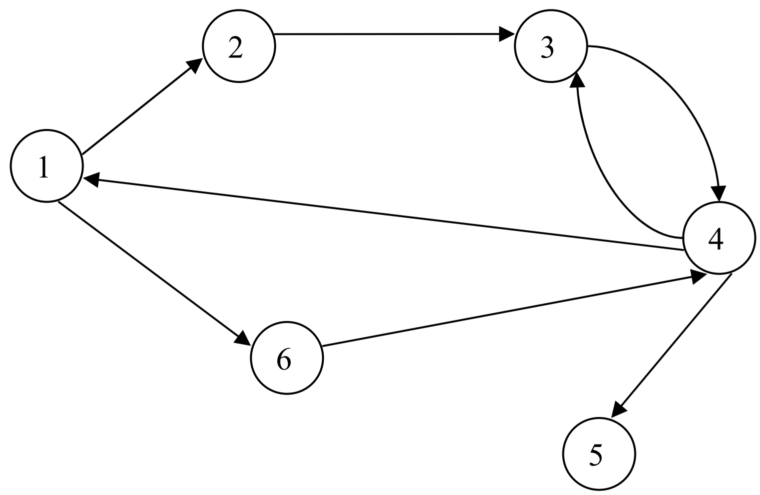


i) List all paths of length 2 starting from vertex 2.

ii) Find a cycle starting at vertex 2.

iii) Draw diagraph of R^2.

25) Let R be a relation whose diagraph is given below :



iv) List all paths of length 3 starting from vertex 3.

v) Find a cycle starting at vertex 6.

vi) Find M_{R^3}.

Unit – II

1 : Questions of 2 marks

1) Define i) a message ii) a word
2) Define i) an (m , n) encoding function ii) an alphabet
3) Define i) a code word ii) a code
4) Define weight of a word. Find the weight of a word 110110101.
5) Define parity check code. If $e : B^4 \rightarrow B^5$ is a parity check code then find e(1010) and e(1011).
6) Define the Hamming distance between the words $x, y \in B^m$. If $e : B^4 \rightarrow B^5$ is a parity check code then find $\delta(e(0110), e(1101))$
7) If $e : B^4 \rightarrow B^5$ is a parity check code then find

   i) $\delta(e(1011), e(1101))$ ii) $\delta(e(0011), e(1001))$

8) Define the minimum distance of an encoding function. If $e : B^2 \to B^4$ is encoding function defined by $e(b_1b_2) = b_1b_2b_1b_2$ then find minimum distance of e.

9) Find the minimum distance of (2 , 3) parity check code.

10) If $e : B^2 \to B^4$ is encoding function defined by $e(b_1b_2) = b_1b_2b_2b_1b_2$, then find minimum distance of e.

11) Define Parity check matrix. If $H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a parity check matrix then find (1,3) group code $e_H : B^1 \to B^3$.

12) Define the minimum distance of a decoding function.

13) Find weight of each of the following words in B^4 : x = 1010 , y = 1110 , z = 0000 , w = 1111. Also find $\delta(x , y)$, $\delta(z , w)$.

14) Find weight of each of the following words in B^7 : x = 1100010, y = 1010110, z = 1111111, w = 1110101. Also find $\delta(x , y)$, $\delta(z , w)$.

15) Compute i) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

16) Compute i) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

17) If $B^m = B \times B \times - - - \times B$ (m factors) is a group under the binary operation $\oplus$ then i) Find the identity element of B^m.

ii) Find inverse of $x \in B^m$. iii) Write the order of B^m.

18) Let e be the (3 , 8) encoding function with minimum distance 3. Let d be the associated maximum likelihood decoding function. Determine the number of errors that (e,d) can correct.

19) Let $H = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ be a parity check matrix. Decode 0101 relative to a maximum likelihood decoding function associate with e_H.

20) Let $H = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ be a parity check matrix. Decode 1101 relative to a maximum likelihood decoding function associate with e_H.

21) If $H = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a parity check matrix then find (2,4) group code $e_H : B^2 \rightarrow B^4$.

22) Define decoding function $d : B^9 \rightarrow B^3$ by $d(y_1y_2y_3y_4y_5y_6y_7y_8y_9) = z_1z_2z_3$, where $z_i = \begin{cases} 1, & \text{if } (y_i, y_{i+3}, y_{i+6}) \text{ has at least two 1's} \\ 0, & \text{if } (y_i, y_{i+3}, y_{i+6}) \text{ has less than two 1's} \end{cases}$, $1 \le i \le 3$.

Determine i) d(101111101) ii) d(100111100).

23) Define decoding function $d : B^9 \rightarrow B^3$ by $d(y_1y_2y_3y_4y_5y_6y_7y_8y_9) = z_1z_2z_3$, where $z_i = \begin{cases} 1, & \text{if } (y_i, y_{i+3}, y_{i+6}) \text{ has at least two 1's} \\ 0, & \text{if } (y_i, y_{i+3}, y_{i+6}) \text{ has less than two 1's} \end{cases}$, $1 \le i \le 3$.

Determine i) d(010000010) ii) d(011000011).

24) Define decoding function $d : B^6 \rightarrow B^2$ by $d(y_1y_2y_3y_4y_5y_6) = z_1z_2$, where $z_i = \begin{cases} 1, & \text{if } (y_i, y_{i+2}, y_{i+4}) \text{ has at least two 1's} \\ 0, & \text{if } (y_i, y_{i+2}, y_{i+4}) \text{ has less than two 1's} \end{cases}$, $1 \le i \le 2$.

Determine i) d(111011) ii) d(010100) iii) d(101011) ii) d(000110).

2 : Multiple choice Questions of 1 marks

1) If e : $B^m \to B^n$ is an encoding function then - - - -

a) m < n and e is onto b) m < n and e is one one

c) m > n and e is onto d) m > n and e is one one

2) If $x \in B^m$ then weight of x is - - - -

a) the number of 0's in x b) the number of 1's in x

c) the difference of the number of 1's and the number of 0's in x

d) m

3) If an encoding function e : $B^m \to B^n$ is a parity check code then - - - - - -

a) m = n + 1 b) n = m + 1 c) m = n d) n = m + m

4) If minimum distance of an encoding function e : $B^m \to B^n$ is k then e can detect - - - -

a) k or fewer errors b) less than k errors

c) more than k errors d) k + 1 errors

5) An encoding function e : $B^m \to B^n$ is a group code if

a) Ran{e} is a subgroup of B^m. b) Ran{e} is a subgroup of B^n.

c) Ran{e} is not a subgroup of B^m. d) none of these

6) If d : $B^n \to B^m$ is a (n,m) decoding function then - - - -

a) $m \leq n$ and d is onto b) $m \leq n$ and d is one one

c) $m \geq n$ and d is onto d) $m \geq n$ and d is one one

7 Let e : $B^m \to B^n$ be an encoding function with minimum distance 2k + 1. If d is maximum likehood decoding function associated with e then [ed] can correct - - - -

a) more than k errors b) more than 2k + 1 errors

c) k errors d) less than or equal to k errors

8) If B = {0 , 1} then order of a group B^4 = - - - -

a) 2 b) 4 c) 8 d) 16

3 : Questions of 3 marks

1) Let x, y be elements of B^m. Show that i) $\delta(x , y) \geq 0$

ii) $\delta(x , y) = 0 \Leftrightarrow x = y$.

2) Let x, y, z be elements of B^m. Show that i) $\delta(x , y) = \delta(y , x)$

ii) $\delta(x , y) \leq \delta(x , z) + \delta(z , y)$.

1) If minimum distance of an encoding function $e : B^m \to B^n$ is at least $k + 1$ then prove that e can detect k or fewer errors.

2) If an encoding function $e : B^m \to B^n$ can detect k or fewer errors then prove that its minimum distance is at least $k + 1$.

3) Let $e : B^m \to B^n$ be a group code. Prove that the minimum distance of e is the minimum weight of a non zero code.

4) Let $m < n$, $n - m = r$ and $x = b_1b_2 - - - - b_mx_1x_2 - - - x_r \in B^n$ and $x * H = \bar{0}$, where H is the parity check matrix of order nxr. Show that there exists an encoding function $e_H : B^m \to B^n$ such that $x = e_H(b)$, for some $b \in B^m$.

5) Consider (3 , 6) encoding function $e : B^3 \to B^6$ defined by e(000) = 000000, e(001) = 001100, e(010) = 010011, e(100) = 100101, e(011) = 011111, e(101) = 101001, e(110) = 110110, e(111) = 111010. Show that e is a group code.

6) Consider (3 , 6) encoding function $e : B^3 \to B^6$ defined by e(000) = 000000, e(001) = 001100, e(010) = 010011, e(100) = 100101, e(011) = 011111, e(101) = 101001, e(110) = 110110, e(111) = 111010. How many errors will e detect?

7) Consider (3 , 8) encoding function e : $B^3 \rightarrow B^8$ defined by e(000) = 00000000, e(001) = 10111000, e(010) = 00101101, e(100) = 10100100, e(011) = 10010101, e(101) = 10001001, e(110) = 00011100, e(111) = 00110001. How many errors will e detect?

8) Consider (3 , 8) encoding function e : $B^3 \rightarrow B^8$ defined by e(000) = 00000000, e(001) = 10111000, e(010) = 00101101, e(100) = 10100100, e(011) = 10010101, e(101) = 10001001, e(110) = 00011100, e(111) = 00110001. Is e a group code? Why?

9) Consider (2 , 6) encoding function e : $B^2 \rightarrow B^6$ defined by e(00) = 000000, e(01) = 011110, e(10) = 101010, e(11) = 111000. Find the minimum distance of e. Is e a group code? Why?

10) Consider (2 , 6) encoding function e : $B^2 \rightarrow B^6$ defined by e(00) = 000000, e(01) = 011110, e(10) = 101010, e(11) = 111000. How many errors will e detect?

11) Let e be (3 , 5) encoding function defined by e(000) = 00000, e(001) = 11110, e(010) = 01101, e(100) = 01010, e(011) = 10011, e(101) = 10100, e(110) = 00111, e(111) = 11001. Show that e is a group code.

12) Let e be (3 , 5) encoding function defined by e(000) = 00000, e(001) = 11110, e(010) = 01101, e(100) = 01010, e(011) = 10011, e(101) = 10100, e(110) = 00111, e(111) = 11001. How many errors will e detect?

13) Let $H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the group code $e_H : B^2 \rightarrow B^5$.

14) Let $H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the group code $e_H : B^3 \rightarrow B^6$.

15) Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the group code $e_H : B^3 \rightarrow B^6$.

16) Consider (3 , 8) encoding function $e : B^3 \rightarrow B^8$ defined by e(000) = 00000000, e(001) = 10111000, e(010) = 00101101, e(100) = 10100100, e(011) = 10010101, e(101) = 10001001, e(110) = 00011100, e(111) = 00110001. Let d be an (8 , 3) maximum likelihood decoding function associate with e. How many errors can (e,d) detect?

17) Consider (3 , 5) encoding function $e : B^3 \rightarrow B^5$ defined by by e(000) = 00000, e(001) = 11110, e(010) = 01101, e(100) = 01010, e(011) = 10011, e(101) = 10100, e(110) = 00111, e(111) = 11001. Let d be an (5 , 3) maximum likelihood decoding function associate with e. How many errors can (e,d) detect?

18) Let e be the (3 , 8) encoding function with minimum distance 4. Let d be the associated maximum likelihood decoding function. Determine the number of errors that (e,d) can correct.

19)Find i) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

20)Explain the procedure for obtaining a maximum likelihood decoding function associated with a group code $e : B^m \rightarrow B^n$.

21)Explain the decoding procedure for a group code given by a parity check matrix.

22)Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Decode 011001 relative to a maximum likelihood decoding function associate with e_H.

23)Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Decode 101011 relative to a maximum likelihood decoding function associate with e_H.

Unit – III

1 : Questions of 2 marks

1) Let (R , +) be a group of real numbers under addition. Show that $f : R \rightarrow R$, defined by $f(x) = 3x$, for all $x \in R$, is a group homomorphism. Find Ker(f).

2) Let (R , +) be a group of real numbers under addition. Show that $f : R \rightarrow R$, defined by $f(x) = 2x$, for all $x \in R$, is a group homomorphism. Find Ker(f).

3) If $(R , +)$ is a group of real numbers under addition and $(R^+ , \cdot)$ is a group of positive real numbers under multiplication. Show that $f : R \to R^+$, defined by $f(x) = e^x$, for all $x \in R$, is a group homomorphism. Find Ker(f).

4) Let $(R^* , \cdot)$ be a group of non zero real numbers under multiplication. Show that $f : R^* \to R^*$, defined by $f(x) = x^3$, for all $x \in R^*$, is a group homomorphism. Find Ker(f).

5) Let $(C^* , \cdot)$ be a group of non zero complex numbers under multiplication. Show that $f : C^* \to C^*$, defined by $f(z) = z^4$, for all $z \in C^*$, is a group homomorphism. Find Ker(f).

6) Let $(Z , +)$ be a group of integers under addition and $G = \{5^n : n \in Z\}$ a group under multiplication. Show that $f : Z \to G$, defined by $f(n) = 5^n$, for all $n \in Z$, is onto group homomorphism.

7) Let $(Z , +)$ and $(E , +)$ be the groups of integers and even integers respectively under addition. Show that $f : Z \to E$, defined by $f(n) = 2n$, for all $n \in Z$, is an isomorphism.

8) Define a group homomorphism. Let $(G , *)$, $(G' , *')$ be groups with identity elements e , e' respectively. Show that $f : G \to G'$, defined by $f(x) = e'$, for all $x \in G$, is a group homomorphism.

9) Let $G = \{a , a^2 , a^3 , a^4 , a^5 = e\}$ be the cyclic group generated by a. Show that $f : (Z_5 , +_5) \to G$, defined by $f(\bar{n}) = a^n$, for all $\bar{n} \in Z_5$, is a group homomorphism. Find Ker(f).

10) Let $f : (R , +) \to (R , +)$ be defined by $f(x) = x + 1$, for all $x \in R$. Is f a group homomorphism? Why?

11) Let $G = \{1 , -1 , i , -i\}$ be a group under multiplication and $Z'_8 = \{\bar{1} , \bar{3} , \bar{5} , \bar{7}\}$ a group under multiplication modulo 8. Show that G and Z'_8 are not isomorphic.

12) Show that the group $(Z_4 , +_4)$ is isomorphic to the group $(Z'_5 , \times_5)$.

13) Let $f : G \to G'$ be a group homomorphism. If $a \in G$ and $o(a)$ is finite then show that $o(f(a)) \mid o(a)$.

14) Let $f : G \to G'$ be a group homomorphism If H' is a subgroup of G' then show that $Ker(f) \subseteq f^{-1}(H')$.

15) Let $f : G \to G'$ be a group homomorphism and $o(a)$ is finite, for all $a \in G$. If f is one one then show that $o(f(a)) = o(a)$.

16) Let $f : G \to G'$ be a group homomorphism and $o(f(a)) = o(a)$, for all $a \in G$. Show that f is one one.

2 : Multiple choice Questions of 1 marks

Choose the correct option from the given options.

1) Every finite cyclic group of order n is isomorphic to - - -

   a) $(Z , +)$ b) $(Z_n , +_n)$ c) $(Z_n , \times_n)$ d) $(Z'_n , \times_n)$

2) Every infinite cyclic group is isomorphic to - - -

   a) $(Z , +)$ b) $(Z_n , +_n)$ c) $(Z_n , \times_n)$ d) $(Z'_n , \times_n)$

3) Let $f : G \to G'$ be a group homomorphism and $a \in G$. If $o(a)$ is finite then - - -

   a) $o(f(a)) = \infty$ b) $o(f(a)) \mid o(a)$.

   c) $o(a) \mid o(f(a))$ d) $o(f(a)) = 0$.

4) A group $G = \{1 , -1 , i , -i\}$ under multiplication is not isomorphic to - - -

   a) $(Z_4 , +_4)$ b) G

   c) $(Z'_8 , \times_8)$ d) none of these.

5) Let $f : G \to G'$ be a group homomorphism. If G is abelian then f(G) is - - -

   a) non abelian b) abelian

   c) cyclic d) empty set

6) Let $f : G \to G'$ be a group homomorphism. If G is cyclic then f(G) is - - -

a) non abelian b) non cyclic

c) cyclic d) finite set

7) A onto group homomorphism $f : G \to G'$ is an isomorphism if Ker(f) = - - -

a) ϕ b) {e} c) $\{e'\}$ d) none of these

8) A function $f : G \to G$, (G is a group) , defined by f(x) = x-1, for all x $\in$ G, is an automorphism if and only if G is - - -

a) abelian b) cyclic c) non abelian d) G = ϕ.

3 : Questions of 4 marks

1) Let $f : G \to G'$ be a group homomorphism . prove that f(G) is a subgroup of G'. Also prove that if G is abelian then f(G) is abelian.

2) Let $f : G \to G'$ be a group homomorphism. Show that f is one one if and only if Ker(f) = {e}.

3) Let G = {1 , -1 , i , -i} be a group under multiplication. Show that f : (Z , +) $\to$ G, defined by f(n) = i^n , for all n $\in$ Z, is onto group homomorphism. Find Ker(f).

4) Let G = {1 , -1 , i , -i} be a group under multiplication. Show that f : (Z , +) $\to$ G, defined by f(n) =$(-i)^n$, for all n $\in$ Z, is onto group homomorphism. Find Ker(f).

5) Let G = $\left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a , b \in R,\ a^2 + b^2 \neq 0 \right\}$ be a group under multiplication and C^* be a group of non zero complex numbers under

multiplication. Show that $f : C^* \to G$ defined by $f(a + ib) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, for all $a + ib \in C^*$, is an isomorphism.

6) Define a group homomorphism. Prove that homomorphic image of a cyclic group is cyclic.

7) Let $f : G \to G'$ be a group homomorphism. Prove that

   i) $f(e)$ is the identity element of G', where e is the identity element of G

   ii) $f(a^{-1}) = (f(a))^{-1}$, for all $a \in G$

   iii) $f(a^m) = (f(a))^m$, for all $a \in G$, $m \in Z$.

8) Let $(C^*, \cdot)$.$(R^*, \cdot)$ be groups of non zero complex numbers, non zero real numbers respectively under multiplication. Show that $f : C^* \to R^*$ defined by $f(z) = |z|$, for all $z \in C^*$, is a group homomorphism. Find Ker(f). Is f onto? Why?

9) Let $(C^*, \cdot)$, $(R^*, \cdot)$ be groups of non zero complex numbers, non zero real numbers respectively under multiplication. Show that $f : C^* \to R^*$ defined by $f(z) = |\bar{z}|$, for all $z \in C^*$, is a group homomorphism. Find Ker(f). Is f onto? Why?

10) Let $G = \{1, -1\}$ be a group under multiplication. Show that $f : (Z, +) \to G$ defined by $f(n) = \begin{cases} 1 & , \text{ if n iseven} \\ -1 & , \text{ if n is odd} \end{cases}$

    is onto group homomorphism. Find Ker(f).

11) Let $(R^+, \cdot)$ be a group of positive reals under multiplication. Show that $f : (R, +) \to R^+$ defined by $f(x) = 2^x$, for all $x \in R$, is an isomorphism.

12) Let $(R^+, \cdot)$ be a group of positive reals under multiplication. Show that $f : (R, +) \to R^+$ defined by $f(x) = e^x$, for all $x \in R$, is an isomorphism.

13) If $f : G \to G'$ is an isomorphism and $a \in G$ then show that $o(a) = o(f(a))$.

14) Prove that every finite cyclic group of order n is isomorphic to $(Z_n, +_n)$.

15) Prove that every infinite cyclic group is isomorphic to $(Z , +)$.

16) Let G be a group of all non singular matrices of order 2 over the set of reals and R^* be a group of all nonzero reals under multiplication. Show that $f : G \to R^*$, defined by $f(A) = | A |$, for all $A \in G$, is onto group homomorphism. Is f one one? Why?

17) Let G be a group of all non singular matrices of order n over the set of reals and R^* be a group of all nonzero reals under multiplication. Show that $f : G \to R^*$, defined by $f(A) = | A |$, for all $A \in G$, is onto group homomorphism.

18) Let R^* be a group of all nonzero reals under multiplication. Show that $f : R^* \to R^*$, defined by $f(x) = | x |$, for all $x \in R^*$, is a group homomorphism. Is f onto? Justify.

19) Prove that every group is isomorphic to it self. If G_1, G_2 are groups such that $G_1 \cong G_2$ then prove that $G_2 \cong G_1$.

20) Let G_1, G_2, G_3 be groups such that $G_1 \cong G_2$ and $G_2 \cong G_3$. Prove that $G_1 \cong G_3$.

21) Show that $f : (C , +) \to (C , +)$ defined by $f(a + ib) = -a + ib$, for all $a + ib \in C$, is an automorphism.

22) Show that $f : (C , +) \to (C , +)$ defined by $f(a + ib) = a - ib$, for all $a + ib \in C$, is an automorphism.

23) Show that $f : (Z , +) \to (Z , +)$ defined by $f(x) = -x$, for all $x \in Z$, is an automorphism.

24) Let G be an abelian group. Show that $f : G \to G$ defined by $f(x) = x^{-1}$, for all $x \in G$, is an automorphism.

25) Let G be a group and $a \in G$. Show that $f_a : G \to G$ defined by $f_a(x) = axa^{-1}$, for all $x \in G$, is an automorphism.

26) Let G be a group and $a \in G$. Show that $f_a : G \to G$ defined by $f_a(x) = a^{-1}xa$, for all $x \in G$, is an automorphism.

27) Let G = {a , a^2 , a^3 , - - - , a^{12} (= e)}be a cyclic group generated by a. Show that f : G → G defined by f(x) = x^4, for all x ∈ G, is a group homomorphism. Find Ker(f).

28)Let G = {a , a^2 , a^3 , - - - , a^{12} (= e)}be a cyclic group generated by a. Show that f : G → G defined by f(x) = x^3, for all x ∈ G, is a group homomorphism. Find Ker(f).

29) Show that f : (C , +) → (R , +) defined by f(a + ib) = a, for all a + ib ∈ C, is onto homomorphism. Find Ker(f).

30) Show that homomorphic image of a finite group is finite. Is the converse true? Justify.

Unit – IV

1 : Questions of 2 marks

1) In a ring (Z , ⊕ , ⊙), where a ⊕ b = a + b – 1 and a ⊙ b = a + b – ab , for all a , b ∈ Z, find zero element and identity element.
2) Define an unit. Find all units in (Z_6 , $+_6$, $\times_6$).
3) Define a zero divisor. Find all zero divisors in (Z_8 , $+_8$, $\times_8$).
4) Let R be a ring with identity 1 and a ∈ R. Show that

   i) (–1)a = –a ii)(–1) (–1) = 1
5) Let R be a commutative ring and a , b ∈ R. Show that $(a – b)^2 = a^2 – 2ab + b^2$.
6) Let (Z[$\sqrt{-5}$] , + , ·) be a ring under usual addition and multiplication of elements of Z[$\sqrt{-5}$]. Show that Z[$\sqrt{-5}$] is a commutative ring . Is 2 + 3$\sqrt{-5}$ a unit in Z[$\sqrt{-5}$]?
7) Let $\overline{m}$ ∈ (Z_n , $+_n$, $\times_n$) be a zero divisor. Show that m is not relatively prime to n, where n > 1.

8) If $\overline{m} \in (Z_n , +_n , \times_n)$ is invertible then show that m and n are relatively prime to n, where n > 1.

9) Let n > 1 and 0 < m < n. If m is relatively prime to n then show that $\overline{m} \in (Z_n , +_n , \times_n)$ is invertible.

10) Let n > 1 and 0 < m < n. If m is not relatively prime to n then show that $\overline{m} \in (Z_n , +_n , \times_n)$ is a zero divisor.

11) Show that a field has no zero divisors.

12)Let R be a ring in which $a^2 = a$, for all $a \in R$. Show that a + a = 0, for all $a \in R$.

13)Let R be a ring in which $a^2 = a$, for all $a \in R$. If $a , b \in R$ and a + b = 0, then show that a = b.

14)Let R be a commutative ring with identity 1. If a , b are units in R then show that a^{-1} and ab are units in R.

15) In $(Z_{12} , +_{12} , \times_{12})$ find (i) $(\bar{3})^2 +_{12} (\bar{5})^{-2}$ (ii) $(\bar{7})^{-1} +_{12} \bar{8}$.

16)In $(Z_{12} , +_{12} , \times_{12})$ find (i) $(\bar{5})^{-1} - \bar{7}$ (ii) $(\overline{11})^{-2} +_{12} \bar{5}$.

2 : Multiple choice Questions of 1 marks

Choose the correct option from the given options.

1) R = {±1, ±2, ±3, - - - } is not a ring under usual addition and multiplication of integers because - - -
   a) R is not closed under multiplication
   b) R is not closed under addition
   c) R does not satisfy associativity w.r.t. addition
   d) R does not satisfy associativity w.r.tmultiplication
2) Number of zero divisors in $(Z_6 , +_6 , \times_6)$ = - - -

a) 0 b) 1 c) 2 d) 3

3) $(Z_{43}, +_{43}, \times_{43})$ is - - -

a) both field and integral domain

b) an integral domain but not a field

c) a field but not an integral domain

d) neither a field nor an integral domain

4) In $(Z_9, +_9, \times_9)$, $\bar{6}$ is - - -

a) a zero divisor b) an invertible element

c) a zero element d) an identity element

5) Every Boolean ring is - - -

a) an integral domain b) a field

c) a commutative ring d) a division ring

6) If a is a unit in a ring R then a is - - -

a) a zero divisor b) an identity element

c) a zero element d) an invertible element

7) If R is a Boolean ring and $a \in R$ then - - -

a) $a + a = a$ b) $a^2 = 0$ c) $a^2 = 1$ d) $a + a = 0$

8) Value of $(\bar{7})^2 - \bar{7}$ in $(Z_8, +_8, \times_8)$ is - - -

a) $\bar{6}$ b) $\bar{4}$ c) $\bar{2}$ d) $\bar{0}$

3 : Questions of 6 marks

1a) Define i) a ring ii) an integral domain iii) a division ring.

b) Show that the set R = {0, 2, 4, 6} is a commutative ring under addition and multiplication modulo 8.

2a) Define i) a commutative ring ii) a field iii) a skew field.

b) In 2Z, the set of even integers, we define a + b = usual addition of a and b and $a \odot b = \frac{ab}{2}$. Show that $(2Z, +, \odot)$ is a ring.

3 a) Define i) a ring with identity element ii) an unit element iii) a Boolean ring.

b) Let (2Z , +) be an abelian group of even integers under usual addition. Show that (2Z , + , ⊙) is a commutative ring with identity 2, where $a \odot b = \frac{ab}{2}$, for all $a , b \in 2Z$.

4) a) Define i) a zero divisor ii) an invertible element iii) a field.

b) Let (3Z , +) be an abelian group under usual addition where $3Z = \{3n \mid n \in Z\}$. Show that (3Z , + , ⊙) is a commutative ring with identity 3, where $a \odot b = \frac{ab}{3}$, for all $a , b \in 3Z$.

5) a) Let (R, +, ·) be a ring and a, b, c ∈ R. Prove that

i) $a \cdot 0 = 0$ ii) $(a – b)c = ac – bc$.

b) Show that (Z , ⊕ , ⊙)is a ring, where $a \oplus b = a + b – 1$ and $a \odot b = a + b – ab$, for all $a , b \in Z$.

6) a) Let (R, +, ·) be a ring and a, b, c ∈ R. Prove that

i) $a \cdot (–b) = –(ab)$ ii) $a (b – c)c = ab – ac$.

b) Show that the abelian group $(Z[\sqrt{-5}] , +)$ is a ring under multiplication $(a + b\sqrt{-5})(c + d\sqrt{-5}) = ac – 5bd + (ad + bc)\sqrt{-5}$.

7) a) Define i) a division ring ii) an unit element iii) an integral domain

b) Show that the abelian group (Z[i] , +) is a ring under multiplication

$(a + bi)(c + di) = ac – bd + (ad + bc) i$, for all $a + bi ,c + di \in Z[i]$.

8a) Let R be a ring with identity 1 and $(ab)^2 = a^2b^2$, for all a, b ∈ R. Show that R is commutative.

b)Show that the abelian group $(Z_n , +_n)$ is a commutative ring with identity $\bar{1}$ under multiplication modulo n operation.

9 a) Show that a ring R is commutative if and only if $(a + b)^2 = a^2 + 2ab + b^2$, for all $a, b \in R$.

b) Show that $Z[i] = \{a + ib \mid a , b \in Z\}$, the ring of Gaussian integers, is an integral domain.

10 a) Show that a commutative ring R is an integral domain if and only if $a , b , c \in R, a \neq 0, ab = ac \Rightarrow b = c$.

b) Prepare addition modulo 4 and multiplication modulo 4 tables. Find all invertible elements in Z_4.

11 a) Show that a commutative ring R is an integral domain if and only if $a , b \in R, ab = 0 \Rightarrow$ either $a = 0$ or $b = 0$.

b) Prepare addition modulo 5 and multiplication modulo 5 tables. Find all invertible elements in Z_5.

12 a) Let R be a commutative ring. Show that the cancellation law with respect to multiplication holds in R if and only if $a , b \in R, ab = 0 \Rightarrow$ either $a = 0$ or $b = 0$.

b) Prepare a multiplication modulo 6 table for a ring $(Z_6 , +_6 , \times_6)$. Hence find all zero divisors and invertible elements in Z_6.

13 a) For $n > 1$, show that Z_n is an integral if and only if n is prime.

b) Let $R = \left\{ \begin{bmatrix} z & w \\ -\overline{w} & \overline{z} \end{bmatrix} : z, w \in C \right\}$ be a ring under addition and multiplication, where $C = \{a + ib \mid a , b \in R\}$. Show that R is a divison ring.

14 a) Prove that every field is an integral domain. Is the converse true? Justify.

b) Which of the following rings are fields? Why?

i) $(Z , + , \times)$ ii) $(Z_5 , +_5 , \times_5)$ iii) $(Z_{25} , +_{25} , \times_{25})$.

15) a) Prove that every finite integral domain is a field.

b) Which of the following rings are integral domains? Why?

i) $(2Z, +, \times)$ ii) $(Z_{50}, +_{50}, \times_{50})$ iii) $(Z_{17}, +_{17}, \times_{17})$.

16 a) Prove that a Boolean ring is a commutative ring.

b) Give an example of a division ring which is not a field.

17 a) for $n > 1$, show that Z_n is a field if and onle if n is prime.

b) Let $R = \{a + bi + cj + dk \mid a, b, c, d \in R\}$, where $i^2 = j^2 = k^2 = -1$, $ij = k = -ji$, $jk = i = -kj$, $ki = j = -ik$. Show that every nonzero element of R is invertible.

18 a) If R is a ring and $a, b \in R$ then prove or disprove $(a + b)^2 = a^2 + 2ab + b^2$.

b) Show that R^+, the set of all positive reals forms a ring under the following binary operations :

$$a \oplus b = ab \text{ and } a \odot b = a^{\log_5 b}, \text{ for all } a,b \in R^+.$$

19 a) Define i) a ring ii) a Boolean ring iii) an invertible element.

b) Let p be a prime and $(pZ, +)$ be an abelian group under usual addition, show that $(pZ, +, \odot)$ is a commutative ring with identity element p where $a \odot b = \frac{ab}{p}$, for all $a, b \in pZ$.

20) a) Define i) a ring with identity element ii) a commutative ring iii) a zero divisor.

b) Show that R^+, the set of all positive reals forms a ring under the following binary operations :

$$a \oplus b = ab \text{ and } a \odot b = a^{\log_7 b}, \text{ for all } a,b \in R^+.$$

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