NORTH MAHARASHTRA UNIVERSITY,

JALGAON

Question Bank

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Class : S.Y. B. Sc. Subject : Mathematics

Paper : MTH – 222 (B) (Numerical Analysis)

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Question Bank

Paper : MTH – 222 (B)

Numerical Analysis

Unit – I

1 : Questions of 2 marks

1) What is meant by "Inherent error"?
2) Define Rounding error.
3) Define Truncation error.
4) Explain : Absolute error and relative error.
5) What is meant by "Percentage error"?
6) State with usual notation the Newton Raphson formula.
7) In the method of false position, state the formula for the first approximation of the root of given equation, where symbol have their usual meaning.
8) Find the root of the equation $x^3 - x - 1 = 0$ lying between 1 and 2 by Bisection method up to first iteration.
9) Show that a real root of the equation $x^3 - 4x - 9 = 0$ lies between 2 and 3 by Bisection method.
10) Using Bisection method, show that a real root of the equation $3x - \sqrt{1+\sin x} = 0$ lies between 0 and 1.
11) Find the first approximation of x for the equation $x = 0.21\sin(0.5+x)$ by iteration method starting with $x = 0.12$.

12) Find an iterative formula to find $\sqrt{N}$ where N is a positive number by Newton Raphson method.

13) Using Newton Raphson method find first approximation x_1 for finding $\sqrt{10}$, taking $x_0 = 3.1$.

14) Using Newton Raphson method find first approximation x_1 for finding $\sqrt[3]{13}$, taking $x_0 = 2.5$.

15) Obtain Newton Raphson formula for finding a r^{th} root of a given number c.

16) Show that a real root of a equation $x\log_{10}x - 1.2 = 0$ lies between 2 and 3.

17) What is meant by significant figure? Find the significant figures in 0.00397.

18) If true value of a number is 36.25and its absolute error is 0.002. find the relative error and percentage error.

19) If the absolute error is 0.005 and relative error is 3.264×10^{-6}, then find the true value and percentage error.

2 : Fill in the blanks/Multiple choice Questions of 1 marks

1) If X is the true value of the quantity and X_1 is the approximate value then the relative error is $E_R =$ - - - - and percentage error is $E_P =$ - - - -

2) If X is the true value and X_1 is the approximate value of the given quantity then its absolute error is $E_A =$ - - - - and relative is error $E_R =$ - - - -

3) Every algebraic equation of the n^{th} degree has exactly - - - - roots.

4) After rounding of the number 2.3762 to the two decimal places, we get the number - - - -.

5) Rounding off the number 32.68673 to 4 significant digits, we get a number - - - -

a) 32.68 b) 32.69 c) 32.67 d) 32.686

6) In bisection method if roots lies between a and b then $f(a)\times f(b)$ is - - - -

a) < 0 b) $= 0$ c) > 0 d) none of these

7) If percentage error of a number is 3.264×10^{-4} then its relative error is - - - -

a) 3.264×10^{-5} b) 3.264×10^{-6}

c) 3.264×10^{-7} d) none of these

8) The root of the equation $x^3 - 2x - 5 = 0$ lies between - - - -

a) 0 and 1 b) 1 and 2 c) 2 and 3 d) 3 and 4

9) In Newton Raphson method for finding the real root of equation $f(x) = 0$, the value of x is given by - - - -

a) $x_0 - \frac{f(x_0)}{f'(x_0)}$ b) x_0 c) $\frac{f(x_0)}{f'(x_0)}$ d) none of these

3 : Questions of 4 marks

1) Explain the Bisection method for finding the real root of an equation $f(x) = 0$.
2) Explain the method of false position for finding the real root of an equation $f(x) = 0$.
3) Explain the iteration method for finding the real root of an equation $f(x) = 0$. Also state the required conditions.

4) State and prove Newton-Raphson formula for finding the real root of an equation f(x) = 0.

5) Explain in brief Inherent error and Truncation error. What is meant by absolute, relative and percentage errors? Explain.

6) Using the Bisection method find the real root of each of the equation :

(i) $x^3 - x - 1 = 0$. (ii) $x^3 + x^2 + x + 7 = 0$.

(iii) $x^3 - 4x - 9 = 0$. (iv) $x^3 - x - 4 = 0$.

(v) $x^3 - 18 = 0$. (vi) $x^3 - x^2 - 1 = 0$.

(vii) $x^3 - 2x - 5 = 0$. (viii) $x^3 - 9x + 1 = 0$.

(ix) $x^3 - 10 = 0$. (x) $8x^3 - 2x - 1 = 0$.

(xi) $3x - \sqrt{1 + \sin x} = 0$. (xii) $x\log_{10}x = 1.2$.

(xiii) $x^3 - 5x + 1 = 0$. (xiv) $x^3 - 16x^2 + 3 = 0$.

(xv) $x^3 - 20x^2 - 3x + 18 = 0$. (up to three iterations).

7) Using Newton-Raphson method, find the real root of each of the equations given bellow (up to three iterations) :

(i) $x^2 - 5x + 3 = 0$ (ii) $x^4 - x - 10 = 0$

(iii) $x^3 - x - 4 = 0$ (iv) $x^3 - 2x - 5 = 0$

(v) $x^5 + 5x + 1 = 0$ (vi) $\sin x = 1 - x$

(vii) $\tan x = 4x$ (viii) $x^4 + x^2 - 80 = 0$

(ix) $x^3 - 3x - 5 = 0$ (x) $x\sin x + \cos x = 0$

(xi) $x^3 + x^2 + 3x + 4 = 0$ (xii) $x^2 - 5x + 2 = 0$

(xiii) $3x = \cos x + 1$ (xiv) $x\log_{10}x - 1.2 = 0$

(xv) $x^5 - 5x + 2 = 0$ (xvi) $x^3 + 2x^2 + 10x - 20 = 0$

8) Using Newton-Raphson method, find the value of each of :

(i) $\sqrt{10}$ (ii) $\sqrt[3]{13}$ (iii) $\sqrt{17}$ (iv) $\sqrt{29}$ (v) $\sqrt[3]{10}$

9) Using Newton-Raphson method, find the real root of each of:

(i) $e^{-x} - \sin x = 0$ (ii) $\log x = \cos x$ (iii) $\log x - x + 3 = 0$

10) Using the method of false position, obtain a real root of each of the equation (up to 3 iteration)

(i) $x^3 + x^2 + x + 7 = 0$ (ii) $x^3 - 4x - 9 = 0$

(iii) $x^3 - 18 = 0$ (iv) $x^3 - x^2 - 1 = 0$

(v) $x^3 - 2x - 5 = 0$ (vi) $x^3 - 9x + 1 = 0$

(vii) $x^3 - x - 1 = 0$ (viii) $x\log_{10}x - 1.2 = 0$

(ix) $\cos x = 3x - 1$ (x) $xe^x = 2$

(xi) $x^3 - x - 4 = 0$ (xii) $x^3 - x^2 - 2 = 0$

(xiii) $xe^x - 3 = 0$ (xiv) $x^2 - \log_e x - 12 = 0$

11) Using the iterative method, find the real root of each of the equation to four significant figures (up to 3 iterations)

(i) $2x - \log_{10}x - 7 = 0$ (ii) $e^{-x} = 10x$

(iii) $x = \operatorname{cosec}x$ (iv) $x = (5 - x)^{1/3}$

(v) $e^x = \cot x$ (vi) $2x = \cos x + 3$

(vii) $x^3 + x^2 - 1 = 0$ (viii) $\cos x = 3x - 1$

(ix) $\sin x = 10(x - 1)$ (x) $x^3 - x^2 - x - 1 = 0$

(xi) $\tan x = x$ (xii) $x = 0.21\sin(0.5+x)$

Unit – II

1 : Questions of 2 marks

1) Define i) forward difference operator ii) backward difference operator. Find $\Delta \tan^{-1}x$.
2) Define shift operator E. Prove that $E = 1 + \Delta$.
3) Define central difference operator δ and prove that $\delta = \Delta E^{-1/2} = \nabla E^{1/2}$.
4) With usual notations prove that $\mu^2 = \frac{1}{4}(\delta^2 + 4)$.

5) Prove that $u_0 - u_1 + u_2 - u_3 + - - - - = \frac{1}{2}u_0 - \frac{1}{4}u_0 + \frac{1}{8}\Delta^2 u_0 - \frac{1}{16}\Delta^3 u_0 +$ - - - - .

6) Given $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 200$, $u_4 = 100$, $u_5 = 8$. Find $\Delta^5 u_0$.

7) Prove that $u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + - - - = e^x[u_0 + \Delta x_0 + \frac{x^2}{2!}\Delta^2 u_0 + - - - -]$

8) State Gauss's forward central difference formula.

9) State Gauss's backward central difference formula.

10) State Lagrange's interpolation formula.

11) Using Lagrange's interpolation formula find u_3 if $u_0 = 580$, $u_1 = 556$, $u_2 = 520$, $u_4 = 385$.

12) Define averaging operator μ. Show that $\mu = \frac{E^{1/2} + E^{-1/2}}{2}$.

13) Show that $\frac{\nabla^2 x^3}{\delta^2 x^2} = 3$.

14) Show that $\delta = E^{1/2} - E^{-1/2}$.

15) Using the method of separation of symbols prove that $u_{x+n} = u_n{}^xC_1 \Delta u_{n-1} + {}^{x+1}C_2 \Delta^2 u_{n-2} + - - - -$

16) Given that $u_0 + u_8 = 1.9243$, $u_1 + u_7 = 1.9590$, $u_2 + u_6 = 1.9823$, $u_3 + u_5 = 1.9956$,. Find u_4 using $\Delta^8 u_0 = 0$.

17) Construct a forward difference table for the following values of x, y :

x	0	5	10	15	20	25
y = f(x)	6	10	13	17	23	21

18) Construct a backward difference table for the following values of x, y

x	10	20	30	40	50
y = f(x)	45	65	80	92	100

19) Prove that $(1 + \Delta)(1 - \nabla) = 1$

20) Find $\left(\frac{\Delta^2}{E}\right)(x^3)$.

21) Prove that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$.

22) Prove that $u_3 = u_2 + \Delta u_1 + \Delta^2 u_0 + \Delta^3 u_0$.

23) Find the difference table for the data given below :

x	0	1	2	3	4
f(x)	3	6	11	18	27

24) Show that $\Delta^n y_x = y_{x+n} - {}^nC_1 y_{x+n-1} + {}^nC_2 y_{x+n-2} + ---- + (-1)^n\ y_x$.

25) Given $u_0 = 1, u_1 = 11, u_2 = 21, u_3 = 28, u_4 = 29$. Show that $\Delta^4 u_0 = 0$.

26) Form the difference table for the data :

x	1	2	3	4
u	21	15	12	10

27) Find $\frac{dy}{dx}$ at (2 , –2) of a curve passing through the points (0 , 2), (2 , –2), (3 , –1) using Lagrange's interpolation formula.

28) Find the value of $\delta^4 y_2$ given below

x	0	1	2	3	4
y	1	2	9	28	65

29 Find the cubic polynomial for y(0) = 1, y(1) = 0, y(2) = 1, y(3) = 10.

2 : Fill in the blanks/Multiple choice Questions of 1 marks

1) $\frac{1}{h}\left(\nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + -----\right) = ----$.

2) The value off $E^{-n}f(x) = ----$

3) The value of $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$ = - - - - -

4) If in a data six values are given and two values are missing then fifth differences are - - - - and sixth differences are - - - -

5) The value of $\left\{\frac{\Delta^2}{E}\right\}x^4$ is = - - - -

6) The value of $\log\left\{1+\frac{\Delta f(x)}{f(x)}\right\}$ is - - - -

7 The Lagrange's interpolation formula is used for the arguments which are - - - - spaced

a) equally b) distinct c) unequally d) none of these

8) $1 + \Delta =$ - - - -

a) E^{-1} b) ∇ c) E d) δ

9) If n value of f(x) are given then $\Delta^n f(x)$ is - - - - -

a) 0 b) 1 c) 2 d) n

10) The technique for computing the value of the function inside the given argument is called - - - -

a) interpolation b) extrapolation

c) partial fraction d) inverse interpolation

3 : Questions of 3 marks

1) For any positive integer prove that $E^n = (1 + \Delta)^n$.

2) Prove that $\Delta^n(x^n) = n!h^n$.

3) State and prove Lagrange's interpolation formula.

4) State and prove the Gauss's forward central difference formula.

5) State and prove the Gauss's backward central difference formula.

6) Find $\Delta\left[\frac{a^x}{(x+1)!}\right]$.

7) Find $\Delta(e^{ax}\sin bx)$.

8) Prove the identity $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + - - - - + \Delta^{n-1}u_{x-n} + \Delta^n u_{x-n}$.

9) Prove that $u_1 + u_2 + u_3 + - - - - + u_n = {}^nC_1u_1 + {}^nC_2u_2\Delta u_1 + - - - - + \Delta^{n-1}u_1$.

10) Prepare a table of forward differences for the function $f(x) = x^3 + 5x - 7$ for x = -1, 0, 1, 2, 3, 4, 5 and obtain f(7).

11) Find the missing figures in the following table

X	1	2	3	4	5	6	7	8
f(x)	1	8	- - - -	64	- - - -	216	343	512

12) Using the Lagrange's formula find f(5) given that f(1) = 2, f(2) = 4, f(3) = 8, f(4) = 16, f(17) = 38.

13) Using the Lagrange's interpolation formula, express the function $\frac{3x^2 + x + 2}{(x-1)(x-2)(x-3)}$ as sums of partial fractions.

14) Show that $e^x = \left(\frac{\Delta^2}{E}\right)e^x\left(\frac{Ee^x}{\Delta^2 e^x}\right)$ the interval of differencing is h.

15) With usual notations, prove that (i) $E = 1 - \Delta$ (ii) $E\nabla = \nabla E$.

16) With usual notations, prove that $\nabla = 1 - E^{-1}$ and $\nabla = E^{-1}\Delta$.

17) Prove that the identity $\Delta^n u_{x-n} = u_x - {}^nC_1u_{x-1} + {}^nC_2u_{x-2} - {}^nC_3u_{x-3} + - - - -$

18) Estimate the missing term in the following data

x	0	1	2	3	4
y	1	3	9	- - - -	81

19) Using Lagrange's interpolation formula find $\sqrt{153}$ from the given values

x	150	152	154	156
$f(x) = \sqrt{x}$	12.247	312.329	12.410	12.490

20) Using Lagrange's interpolation formula find $\log_{10}658$, given that $\log_{10}654 = 2.8156$, $\log_{10}656 = 2.8159$, $\log_{10}659 = 2.8189$, $\log_{10}661 = 2.8202$.

21) Find the function from given data :

x	3	2	1	–1
f(x)	–1	8	11	–25

22) Using Gauss's forward formula find y for x = 30 given that

x	21	25	29	33	37
y	18.4708	17.8144	17.1070	16.3432	15.5154

23) Given that $\sqrt{12500} = 111.803399$, $\sqrt{12510} = 111.848111$, $\sqrt{12520} = 111.892806$, $\sqrt{12530} = 111.937483$. Show that $\sqrt{12516} = 111.874930$ by Gauss's backward formula.

24) Prove that $\Delta = \dfrac{\delta^2}{2} + \delta\sqrt{1+\dfrac{\delta^2}{4}}$.

25) Using Gauss's forward formula, find f(x) when x = 3.75 with the help of following data

x	2.5	3	3.5	4	4.5	5
f(x)	24.145	22.043	20.225	18.644	17.262	16.047

26) Using Gauss's backward formula, find the population in the year 1936, given that

Year	1901	1911	1921	1931	1941	1951
Population in thousand	12	15	20	27	39	52

27) Apply Gauss's forward formula to find a polynomial of degree 4 or less which takes the following value of the function u_x.

x	1	2	3	4	5
u_x	1	1	1	1	1

28) Given that

x	0	1	2	3	4
y	1	2	9	28	65

Find the value of $\mu^4 y_2$.

29) Apply Gauss's forward formula to obtain f(32) given that f(25) 0.2707, f(30) = 0.3027, f(35) = 0.3386, f(40) = 0.3794.

30) Find the value of cos51^0,42' by Gauss's backward formula, given that

x	50^0	51^0	52^0	53^0	54^0
Cosx	0.6428	0.6243	0.6157	0.6018	0.5878

31) Prove that $1 + \delta^2\mu^2 = \left(1+\frac{\delta^2}{2}\right)^2$.

Unit – III

1 : Questions of 2 marks

1) State normal equations for fitting a straight line y = ax + b.
2) State normal equations for fitting a second degree polynomial y = a + bx + cx^2.
3) How we reduce the problem of fitting the curve $y = ae^{bx}$ for finding a least square straight line through the given data.
4) How we reduce the problem of fitting a power function $y = ax^c$ for finding a least square straight line through the given data.
5) What is meant by curve fitting? Which method is most useful for this?
6) What is the use of the method of least squares?
7) Find the normal equations for fitting the curve $y = ax^c$.
8) Find the normal equations for fitting the curve $y = ae^{bx}$.

9) For the following data, find $\sum x_i^2, \sum x_i, \sum y_i, \sum x_i y_i$.

x	0	1	2
y	1	6	17

10) For data given below find $\sum x_i^2 y_i$

x	0	1	2	3	4
y	1	0	3	10	21

11) For the following data , find $\sum \log y_i$:

x	1	1.2	1.4	1.6
y	40.17	73.196	133.372	243.02

12) If $a = \log c$ and normal equations of $y = ce^{dx}$ are $10a + 30b = 30.7134$ and $4a + 10b = 13.1991$ then find c.

13) For the following data find the normal equations for fitting a straight line $y = a + bx$.

x	1	2
y	5	8

14) Fit a second degree parabola $y = a + bx + cx^2$ to the data

x	1	2
y	2	5

15) Find the normal equation for fitting the curve $y = a + bx + cx^2$, where given data is as

x	1	2
y	0	3

16) Fit a straight line $y = ax + b$ to the data

x	0	1
y	1	0

17) Fit a straight line $y = ax + b$ to the data

x	0	1
y	2	5

18) If the normal equations for fitting a straight line y = ax + b are 26 = 4a + 6b and 54 = 6a + 4b then fit the above straight line.

19) Fit the second degree equation $y = a + bx + cx^2$ if their normal equations are 35 = 5a + 10b + 30c , 120 = 10a + 30b + 100c, 438 = 30a + 100b + 354c.

20) Fit the parabola $y = a + bx + cx^2$ if their normal equations are 9a + 60c = 11, 60b = 51 and 60a + 708c = –9.

21) Fit the curve $y = ae^{bx}$ if their normal equations are 13.1991 = 4a + 10b and 30.7134 = 10a + 30b.

2 : Fill in the blanks/ Multiple choice Questions of 1 marks

1) The problem of fitting a power function $y = ax^c$ is nothing but the problem of fitting a - - - - by - - - - method.
2) The problem of fitting a curve $y = ae^{bx}$ is the problem of fitting the - - - - by - - - - method.
3) The straight line y = a +bx is fitted to the data by - - - - method and - - - - equations can be solved for two unknowns a and b.
4) One of the normal equations for fitting the straight line y = a + bx is given by $\sum x_i y_i$ = - - - -
5) One of the normal equations for fitting the parabola $y = a + bx + cx^2$ is $\sum x_i^2 y_i$ = - - - -
6) The normal equation for fitting of a straight line y = a + bx is $\sum y_i$ = - - - - - -

a) $na + b\sum x_i$ b) $n^2a + b\sum x_i^2$

c) $na + b\sum x_i^2$ d) $a + b\sum x_i$

7) The normal equation for fitting of a straight line y = a + bx + cx^2 is $\sum x_i y_i$ = - - - - - -

a) $a\sum x_i + b\sum x_i^2 + c\sum x_i^3$ b) $a\sum x_i + b\sum x_i^3 + c\sum x_i^4$

c) $a\sum x_i^2 + b\sum x_i + c\sum x_i^3$ d) $a\sum x_i + b\sum x_i^3 + c\sum x_i^2$

8) The method of - - - - is the most systematic procedure to fit a unique curve from given data

a) least squares b) least cube c) square d) none of these

9) - - - -means to form an equation of the curve from the given data

a) least b) square c) curve fitting d) none of these

10) From the data

x	0	1	2	3	4
y	1	0	3	10	21

$\sum x_i^2$ = - - - - -

a) 12 b) 13 c) 14 d) 6

3 : Questions of 4 marks

1) Explain the least square method for fitting a curve.
2) Explain the method of least squares for fitting a straight line y = a + bx to the given data.
3) Explain how to fit a second degree polynomial y = a + bx + cx^2 by using the method of least squares to the given data.
4) Explain how we fit a power function y = ax^b to the given data by using least square method.

5) Explain how we fit an exponential function $y = ae^{bx}$ to the given data by using the method of least squares.

6) Use the method of least squares to fit the straight line y = a + bx to the data given below

X	0	1	2	3	4
Y	1	2.9	4.8	6.7	8.6

7) Use the method of least squares to fit the straight line y = a + bx to each of the data given below

i)

x	0	1	2	3
y	2	5	8	11

ii)

x	1	2	3	4	6	8
y	2.4	3.1	3.5	4.2	5	6

iii)

x	1	2	3	5	6	8	9
y	2	5	7	10	12	15	19

iv)

x	1	2	3	4	5	6
y	1200	900	600	200	110	50

v)

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

vi)

x	–3	–1	1	4	5	7	10
y	–2	–1	0	1.5	2	3	4.5

vii)

x	6	8	10	12	14	16	18	20	22	24
y	3.8	3.7	4	3.9	4.3	4.2	4.2	4.4	4.5	4.5

8) The temperature T (in ^{0}c) and length l (in mm) of a heated rod is given. If l = a + bt find the best value of a and b for each data :

i)

T	20^0	30^0	40^0	50^0	60^0	70^0
l	600.1	600.4	600.6	600.7	600.9	601.0

ii)

T	10^0	30^0	50^0	70^0	90^0	110^0
l	200.1	200.3	200.5	200.7	200.9	201.1

iii)

T	20^0	40^0	60^0	80^0	100^0	120^0
l	100	200	300	350	400	500

9) The following table gives temperature T (in ^{0}c) and length l (in mm) of a heated rod. If l = a + bt, find the best value of a and b by using least square method

T	20^0	30^0	40^0	50^0	60^0	70^0
l	800.3	800.4	800.6	800.7	800.9	801.0

10) If the straight line y = a +bx is the best fit to the set of points (x_1, y_1), (x_2, y_2), - - - - -, (x_n, y_n). then show that $\begin{vmatrix} x & y & 1 \\ \sum x_i & \sum y_i & n \\ \sum x_i^2 & \sum y_i^2 & \sum x_i \end{vmatrix} = 0$ for i = 1, 2, - - - - , n.

10)Find the value of a, b and c so that $y = a + bx + cx^2$ is the best fitting of each of the data given below :

i)

x	0	1	2	3	4
y	1	0	3	10	21

ii)

x	0	1	2
y	1	6	17

iii)

x	0	1	2	3
y	1	6	17	34

iv)

x	0.78	1.56	2.34	3.12	3.81
y	2.5	1.2	1.12	2.25	4.28

v)

x	1929	1930	1931	1932	1933
y	352	356	357	358	360

vi)

x	1	1.5	2	2.5	3	3.5	4
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

vii)

x	1	1.5	2	2.5	3	3.5	4
y	1.1	1.2	1.5	2.6	2.8	3.3	4.1

viii)

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

ix)

x	0.78	1.56	2.34	3.12	3.81
y	2.5	1.2	1.12	2.25	4.28

x)

x	0	1	2	3
y	1	6	17	34

12) Fit the power function $y = ax^b$ to each of the data given below :

i)

x	1	2	3	4
y	60	30	20	15

ii)

x	2	4	7	10
y	43	25	18	13

iii)

x	2.2	2.7	3.5	4.1
y	65	60	53	50

iv)

x	1	2	3	4	5
y	15.3	20.5	27.4	36.6	49.1

v)

x	0.5	1	1.5	2	2.5	3
y	1.62	1	0.75	0.62	0.52	0.46

vi)

x	1	2	3	4	5
y	1290	900	600	200	110

13) Fit the exponential function $y = ae^{bx}$ for each of the data given below :

i)

x	1	1.2	1.4	1.6
y	40.17	73.196	133.372	243.02

ii)

x	1	2	3	4
y	60	30	20	15

iii)

x	0	0.5	1	1.5	2	2.5
y	0.1	0.45	2.15	9.5	40.35	180.75

iv)

x	2.2	2.7	3.5	4.1
y	65	60	53	50

v)

x	1	2	3	4	5	6
y	1.5	4.6	13.9	40.1	125.1	299.5

vi)

x	1	2	3	4	5	6
y	15.3	20.5	27.4	36.6	49.1	65.6

vii)

x	1	2	3	4	5	6	7	8
y	15.3	20.5	27.4	36.6	49.1	65.6	87.8	117.6

14) Determine the constants a and b for $y = ae^{bx}$ for the following data by least squares method

x(Temperature)	77	100	185	239	285
y(Solubility)	2.4	3.4	7	11.1	19.6

Unit – IV

1 : Questions of 2 marks

1) State the Taylor's series for y(x) at $x = x_0$ if y(x) is the exact solution of $y' = f(x,y)$ with $y(x_0) = y_0$.
2) State the Euler's general formula for $y' = f(x,y)$ with $y(x_0) = y_0$.
3) What is the difference between Euler's method and Euler's modified method.

4) State the Runge-Kutta second order formulae.

5) State the Runge-Kutta fourth order formulae.

6) Which method is more useful in solving the differential equation $y' = f(x,y)$ with $y(x_0) = y_0$?

7) State the iteration formula for Euler's modified method, where $y' = f(x,y)$ with $y(x_0) = y_0$.

8) Given that $\frac{dy}{dx} = y - x$ with $y(0) = 2$. Find K_1 and K_2 .

9) Given that $\frac{dy}{dx} = xy^{1/3}$ with y(1 = 1 Find K_1 and K_2 .

10) Given that $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y(0) = 1$ and $h = 0.1$ Find $y(0.1)$ by Eulers method.

11) Given $y' = x^2 + y$ with $y(0) = 1$ and 0.1. Find $y(0.1)$ by Euler's modified method.

12) Given $y' = x + \sqrt{y}$ with $y(0) = 1$ and 0.2. Find $y(0.2)$ by Euler's modified method.

13) Given $y' = y^2 - x^2$ with $h = 0.1$and $y(0) = 1$, Find $y(0.1)$.

14) Find K_1 and K_2 by Runge-Kutta forth order formulae where $y' = 3x + \frac{y}{2}$ with $y(0.1) = 1$ and $h = 0.1$.

15) Find $y(x)$ if $y' = x + y$, with $y(0) = 1$, $x \in [0 , 1]$ by Taylor's series expansion.

16) Given that $y' = \frac{y-x}{y+x}$ with $y(0) = 1$ and $h = 0.025$ compute $y(0.05)$ using Euler's method.

17) Given that $y' = -2y$ with $y(0) = 1$ and $h = 0.1$, compute $y(0.2)$ using Euler's method.

18) Determine the value $y(0.05)$ by Euler's modified method, given that $y' = y + x^2$ with $y(0) = 1$ and $h = 0.05$.

19) Determine the value $y(0.01)$ using Euler's modified method, given that $y' - y - x^2 = 0$ with $y(0) = 1$ and $h = 0.01$.

20) Given that $\frac{dy}{dx} = x + y$ with $y(0) = 1$ and $h = 0.1$, compute $y(0.1)$ by Runge-Kutta fourth order formulae.

21) Using Runge-Kutta fourth order formulae, compute $y(0.2)$, given that $y' + y^2 = x$ with $y(0) = 1$ and $h = 0.2$.

22) Compute $y(0.1)$ by Runge-Kutta second order formulae, given that $y' = y - x$ with $y(0) = 2$ and $h = 0.1$.

2 : Fill in the blanks/ Multiple choice Questions of 1 marks

1) If the exact solution of equation $y' = f(x,y)$ with $y(x_0) = y_0$ then Taylor's series expansion for $y(x)$ about the point $x = x_0$ is $y(x) =$ - - - -

   a) $y_0 + xy_0' + x^2 y_0'' + \text{- - -}$ b) $y_0 + h^2y_0' + h^2y_0'' + h^3y_0''' + \text{- - -}$

   c) $y_0 + hy_0' + h(y_0'')^2 + \text{- - -}$ d) none of these

2) There is a class of methods as - - - - which do not require the calculations of higher order derivatives and give greater accuracy.

   a) Euler's method b) Euler's modified method

   c) kutta d) Runge-Kutta of second order

3) Runge-Kutta method of second order is the - - - - method

   a) Euler's method b) Taylor's method

   c) Euler's modified d) none of these

4) For $y' = y + x$ with $y(0) = 1$ and $h = 0.1$ the value of K_1 in Runge-Kutta fourth order method is - - - -

   a) 0.1 b) 1.0 c) 0.01 d) 0.11

5) In Runge-Kutta fourth order method K_4 = - - - -

a) $hf(x_1 + h , y_1 + K_3)$ b) $hf(x_1 + h , y_1 + K_2)$

c) $hf(x_1 + h , y_1 + K_1)$ d) $f(x_1 + h , y_1 + K_3)$

6) In Runge-Kutta second order method K_2 = - - - -

a) $f(x_0 + h , y_0 + K)$ b) $f(x_0 + h , y_0 + K_1)$

c) $hf(x_0 + h , y_0 + K_1)$ d) $hf(x_0 + h , y_0 + K_2)$

7) In Euler's method , y_{n+1} = - - - -

a) y_n b) $y_n + f(x_n , y_n)$

c) $y_n + hf(x_n , y_n)$ d) none of these

8) The iteration formula for Euler's modified method is $y_1^{n+1} = y_0 +$ - - - -

a) $f(x_0 , y_0) + h$ b) $\frac{h}{2}[f(x_0 , y_0) + f(x_1 , y_1)]$

c) $\frac{h}{2}[f(x_0 , y_0) + f(x_1 , y_1^{(n)})]$ d) none of these

9) Taylor's series method is the - - - -

a) boundary value problem b) initial value problem

c) valued problem d) none of these

10) The value of $y_1(0)$ i.e. $y(0.05)$ is - - - -when $y' = x^2 + y$ with $y(0) =- 1$ and $h = 0.05$.

a) 1.5 b) 1.05 c) 1.052 d) 1.0525

3 : Questions of 6 marks

1) Explain the method of finding the solution of the differential equation $y' = f(x,y)$ with initial condition $y(x_0) = y_0$ by Taylor's series method.
2) Describe the Euler's method of finding the solution of differential equation $y' = f(x,y)$ with $y(x_0) = y_0$.
3) Describe the Euler's modified method of finding the solution of differential equation $y' = f(x,y)$ with $y(x_0) = y_0$.

4) Why Runge-Kutta methods are more effective in finding the solution of differential equation $y' = f(x,y)$ with $y(x_0) = y_0$. Explain Runge-Kutta second order formulae.
5) State the Runge-Kutta method fourth order formulae for finding the solution of differential equation $y' = f(x,y)$ with $y(x_0) = y_0$. Find y(0.1) by Runge-Kutta second order formulae where $y' = y - x$ with $y(0) = 2$ and $h = 0.1$.
6) Using the Taylor's series for y(x), find y(0.1) correct to four decimal places if y(x) satisfies $y' = x + (-y^2)$ with $y(0) = 1$.
7) Solve the differential equation $y' = x + y$ with $y(0) = 1$, $x \in [0 , 1]$ by Taylor's series expansion to obtain y for $x = 0.1$.
8) Using Taylor's series expansion, find the solution of the differential equation $y' = (0.1) (x^3 + y^2)$ with $y(0) = 1$ correct to 4 decimal places.
9) Using Taylor's method, obtain y(1.3) if the differential equation is $y' = x^2 + y^2$ with $y(1) = 0$.
10) Using Taylor's method, obtain y(0.1) given that $y' = xy + 1$ with $y(0) = 1$.
11) Using Taylor's method, obtain y(4.1) and y(4.2), given that $y' = \frac{1}{x^2 + y}$ with $y(4) = 4$.
12) Obtain the Taylor's series for the differential equation $y' = y\sin x + \cos x$ with $y(0) = 0$.
13) Find y(1.2) by Taylor's series for y(x) given that $y' = x + y$ with $y(1) = 0$.
14) Using Euler's method, solve the differential equation $y' = \frac{y - x}{y + x}$, $y(0) = 1$, find y(0.1) in 4 steps.
15) Using Euler's method, find y(0.5), given that $y' = y^2 - x^2$ with $y(0) = 1$ and $h = 0.1$.
16) Using Euler's method, find y(1.5), given that $y' = xy$ with $y(1) = 5$ in the interval [1 , 1.5] and $h = 0.1$.

17) Using Euler's method, find y(0.2), y(0.4) given that $y' = \frac{y - x}{y + x}$ with y(0) = 1 and h = 0.1.

18) Use Euler's method for each to compute

i) y(0.1) and y(0.2), given that $y' + 2y = 0$ with y(0) = 1 and h = 0.1.

ii) y(0.1) and y(0.2), given that $y' = 1 + y^2$ with y(0) = 1 and h = 0.1.

iii) y(0.02) and y(0.03), given that $y' = -y$ with y(0) = 1 and h = 0.01.

iv) y(0.4) and y(0.6), given that $y' = x + y$ with y(0) = 0 and h = 0.2.

v) y(0.4) given that $y' = xy$ with y(0) = 1 and h = 0.1.

vi) y(2), given that $y' = \sqrt{xy} + 2$ with y(1) = 1 and h = 0.1.

vii) y(0.5), given that $y' = x^2 + y^2$, with y(0) = 0 and h = 0.1.

19) Use Euler's method for each to compute

i) y(0.2) and y(0.4), given that $y' = x + \sqrt{xy}$ with y(0) = 1 and h = 0.2.

ii) y(0.5) and y(0.1), given that $y' = x + y$ with y(0) = 1 and h = 0.05.

iii) y(0.2), given that $y' = \log_{10}(x + y)$ with y(0) = 1 and h = 0.2.

iv) y(0.1), given that $y' = x^2 + y$ with y(0) = 1 and h = 0.05.

v) y(0.02) and y(0.04), given that $y' = x^2 + y$ with y(0) = 1 and h = 0.01.

20) Using Runge-Kutta second order formulae compute y(0.1) and y(0.2) correct to four decimal places, given that $y' = y - x$ with y(0) = 2 and h = 0.1.

21) Using Runge-Kutta fourth order formulae compute

i) y(0.1), given that $y' = 3x + \frac{y}{2}$ with y(0) = 1at x = 0.1 and h = 0.1.

ii) y(0.2), given that $y' = -xy$ with y(0) = 1 and h = 0.2.

iii) y(0.2), with $y' = x + y$ with y(0) = 1 and h = 0.1.

iv) y(1.1), given that $y' = xy^{1/3}$ with y(1) = 1 and h = 0.1.

v) y(0.4), given that $y' = -2xy^2$ with y(0) = 1 and h = 0.2.

vi) y(0.1), given that $y' = y - x$ with y(0) = 2 and h = 0.1.

vii) y(0.2) and y(0.4), given that $y' = 1 + y^2$ with y(0) = 0 and h = 0.2.

viii) y(1), given that $y' = \frac{y - x}{y + x}$ with y(0) = 1 and h = 0.05.

ix) y(1.4), given that $y' = xy$ with y(1) = 2 and h = 0.2.

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