

North Maharashtra university, Jalgaon.

Question Bank

New Syllabus w.e.f. June 2008

Class: S. Y. B. Sc.

Subject: Mathematics

Paper MTH: 222(A).

Laplace Transform

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UNIT I
LAPLACE TRANSFORMS

I. Objective, Short answer questions of 2 marks each.

1. Define Laplace transform of function $F(t)$, $t > 0$.
2. Find $L\{1\}$.
3. Find $L\{t\}$.
4. Find $L\{e^{kt}\}$.
5. Define function of exponential order.
6. Find $L\{F(t)\}$ if $F(t) = \begin{cases} 5, & 0 < t < 3 \\ 0, & t > 3 \end{cases}$
7. Find $L\{F(t)\}$ if $F(t) = \begin{cases} 4, & 0 < t < 1 \\ 3, & t > 1 \end{cases}$
8. Find $L\{\sin 2t - 5 \cos 2t\}$.
9. Find $L\{(\sin t - 5 \cos t)^2\}$.
10. Find $L\{2e^{3t} - e^{-3t}\}$.
11. Evaluate $L\{e^{-t} \cos 2t\}$.
12. If $L\{F(t)\} = f(s)$ then show that $L\{e^{at} F(t)\} = f(s - a)$.
13. If $L\{F(t)\} = f(s)$ then show that $L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$.
14. Evaluate $L\{(t + 2)^2 e^t\}$.
15. Evaluate $L\{t \cosh 3t\}$.
16. Find $L\{4e^{5t} + 6t^3 - 3 \sin 4t + 2 \cos 2t\}$.
17. Find $L\{t^3 e^{-3t}\}$.

18. Show that $F(t) = t^2$ is of exponential order 3.

II. Multiple choice questions of 1 mark each. Choose correct answer.

1. $L\{t^n\}$ is

(a) $\frac{s^{n+1}}{(n)!}$ (b) $\frac{s^{n+1}}{(n+1)!}$ (c) $\frac{n!}{s^{n+1}}$ (d) $\frac{n!}{s^n}$

2. $L\{e^{kt}\}$ is

(a) $\frac{1}{s+k}$ (b) $\frac{k}{s+k}$ (c) $\frac{s}{s-k}$ (d) $\frac{1}{s-k}$

3. $L\{\sin kt\}$ is

(a) $\frac{k}{s^2+k^2}$ (b) $\frac{k}{s+k}$ (c) $\frac{s}{s^2+k^2}$ (d) $\frac{s}{s^2-k^2}$

4. $L\{\cos kt\}$ is

(a) $\frac{s}{s^2+k^2}$ (b) $\frac{k}{s^2+k^2}$ (c) $\frac{1}{s^2+k^2}$ (d) $\frac{s}{s^2-k^2}$

5. $L\{\sinh kt\}$ is

(a) $\frac{k}{s^2+k^2}$ (b) $\frac{k}{s^2-k^2}$ (c) $\frac{s}{s^2-k^2}$ (d) $\frac{1}{s^2+k^2}$

6. $L\{\cosh kt\}$ is

(a) $\frac{s}{s^2+k^2}$ (b) $\frac{s}{s^2-k^2}$ (c) $\frac{k}{s^2+k^2}$ (d) $\frac{k}{s^2-k^2}$

7. If $L\{F(t)\} = f(s)$ then $L\{e^{at}F(t)\}$ is

(a) $f(s+a)$ (b) $f(s^2+a^2)$ (c) $f(s-a)$ (d) $f\left(\frac{s}{a}\right)$

8. If $L\{F(t)\} = f(s)$ then $L\{F(2t)\}$ is

(a) $\frac{1}{2}f\left(\frac{s}{3}\right)$ (b) $\frac{1}{2}f\left(\frac{s}{2}\right)$ (c) $2f\left(\frac{s}{2}\right)$ (d) $\frac{1}{2}f(2s)$

9. $L\{t^4\}$ is

(a) $\frac{12}{s^4}$ (b) $\frac{24}{s^5}$ (c) $\frac{4!}{s^4}$ (d) $\frac{12}{s^5}$

III. Questions of 4 marks each.

1. Show that $L\{t^n\} = \frac{n!}{s^{n+1}}$, $s > 0$, ($t > 0$, $n > -1$).

2. Prove that (i) $L\{\sin kt\} = \frac{k}{s^2 + k^2}$

(ii) $L\{\cos kt\} = \frac{s}{s^2 + k^2}$ if $s > 0$

3. Prove that (i) $L\{\sinh kt\} = \frac{k}{s^2 - k^2}$

(ii) $L\{\cosh kt\} = \frac{s}{s^2 - k^2}$ if $s > |k|$

4. State and prove sufficient condition for existence of Laplace Transform.

5. Find $L\{F(t)\}$ if $F(t) = \begin{cases} e^t, 0 < t \leq 1 \\ 0, t > 1 \end{cases}$

6. Show that (i) $F(t) = t^2$ is of exponential order 3.

(ii) $F(t) = e^{t^3}$ is not of exponential order.

7. Find $L\{F(t)\}$, where $F(t) = \begin{cases} 2t, 0 \leq t \leq 5 \\ 1, t > 5 \end{cases}$

8. Find $L\{F(t)\}$ if $F(t) = \begin{cases} e^t, 0 < t < 5 \\ 3, t > 5 \end{cases}$

9. Find $L\{F(t)\}$ if $F(t) = \begin{cases} t, & 0 < t < 4 \\ 5, & t > 4 \end{cases}$

10. State and prove linearity property for Laplace Transform.

11. Find Laplace Transform of following functions. Specify the value of s for which the Laplace Transform exists.

(i) $(\sin 2t - 5 \cos 2t)$ (ii) $(3 \cosh 5t - 3 \sinh 5t)$

12. Evaluate (i) $L\{(5e^{2t} - 3)^2\}$ (ii) $L\{4 \cos^2 2t\}$

13. Evaluate (i) $L\{\cosh^2 4t\}$ (ii) $L\{\sinh^2 3t\}$

14. Evaluate (i) $L\{e^{2t}(3 \sin 4t - 4 \cos 4t)\}$ (ii) $L\{e^{-t}(3 \sinh 2t - 5 \cosh 2t)\}$

15. Find (i) $L\{e^{-t} \sin^2 t\}$ (ii) $L\{(1 + te^t)^2\}$.

16. If $L\{F(t)\} = f(s)$ and

$$G(t) = \begin{cases} F(t-a), & t > a \\ 0, & t < a \end{cases}$$

prove that $L\{G(t)\} = e^{-as} f(s)$

17. Find $L\{F(t)\}$ if $F(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right), & t > \frac{2\pi}{3} \\ 0, & t < \frac{2\pi}{3} \end{cases}$

18. Find $L\{F(t)\}$ if $F(t) = \begin{cases} (t-1)^2, & t > 1 \\ 0, & 0 < t < 1 \end{cases}$

19. If $L\{F(t)\} = \frac{s^2 - s + 1}{(2s + 1)^2(s - 1)}$ find $L\{F(2t)\}$.

20. Given that $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$ find $L\left\{\frac{\sin 4t}{t}\right\}$

21. Find $L\{F(t)\}$ if

$$(i) F(t) = 2e^{3t} - e^{-3t} \quad (ii) F(t) = (t^2 + 1)^2$$

22. Evaluate (i) $L\{e^{-t} \cos 2t\}$ (ii) $L\{2e^{3t} \sin 4t\}$

23. Evaluate (i) $L\{(t+2)^2 e^t\}$ (ii) $L\{e^{-4t} \cosh 2t\}$.

24. Find $L\{F(t)\}$ if (i) $F(t) = \begin{cases} \sin\left(t - \frac{\pi}{3}\right), & t > \frac{\pi}{3} \\ 0, & t < \frac{\pi}{3} \end{cases}$

$$(ii) F(t) = \begin{cases} \sin\left(t - \frac{2\pi}{3}\right), & t > \frac{2\pi}{3} \\ 0, & t < \frac{2\pi}{3} \end{cases}$$

25. By change of scale property find

$$(i) L\{\cos 5t\} \quad (ii) L\{\sinh 3t\}$$

26. Given that $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$ find $L\left\{\frac{\sin at}{t}\right\}$

27. If $L\{F(t)\} = f(s)$, then prove that $L\{F'(t)\} = sf(s) - F(0)$, by assuming

required conditions. Hence show that $L\{t\} = \frac{1}{s^2}$.

28. If $L\{F(t)\} = f(s)$, then prove that $L\{F''(t)\} = s^2 f(s) - sF(0) - F'(0)$, by

assuming required conditions. Hence show that $L\{\sin kt\} = \frac{k}{s^2 + k^2}$.

29. Given $F(t) = \begin{cases} 2t, & 0 \leq t \leq 1 \\ 1, & t > 1 \end{cases}$

(i) Find $L\{F(t)\}$ (ii) Find $L\{F'(t)\}$ (iii) Does the result

$L\{F'(t)\} = sf(s) - F(0)$ hold? Explain.

30. If $L\{F(t)\} = f(s)$, then prove that $L\left\{\int_0^t F(u)du\right\} = \frac{1}{s} f(s)$. Hence evaluate

$$L\left\{\int_0^t \sin 2udu\right\}.$$

31. Verify directly that $L\left\{\int_0^t (u^2 - u + e^{-u})du\right\} = \frac{1}{s} L\{t^2 - t + e^{-t}\}$.

32. If $L\{F(t)\} = f(s)$, then prove that

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s) = (-1)^n f^n(s).$$

33. Find (i) $L\{t \sin kt\}$ (ii) $L\{t^2 \cos kt\}$

34. Find (i) $L\{t(3 \sin 2t - 2 \cos 2t)\}$ (ii) $L\{t^2 e^{2t}\}$

35. Find (i) $L\{t \cosh 3t\}$ (ii) $L\{t \sinh 2t\}$

36. Show that $\int_0^{\infty} t e^{-3t} \sin t dt = \frac{3}{50}$

37. If $L\{F(t)\} = f(s)$, then prove that $L\left\{\frac{F(t)}{t}\right\} = \int_s^{\infty} f(u)du$, provided

$$\lim_{t \rightarrow 0} \frac{F(t)}{t} \text{ exists hence show that } L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$$

38. Show that $\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$

39. Show that $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\} = \log\left(\frac{s+b}{s+a}\right)$

40. Show that $L\left\{\frac{\cos at - \cos bt}{t}\right\} = \frac{1}{2} \log\left(\frac{s^2 + b^2}{s^2 + a^2}\right)$

41. Show that $\int_0^{\infty} \frac{e^{-3t} - e^{-6t}}{t} dt = \log 2$

42. Evaluate $\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} dt$

43. If $L\{F(t)\} = f(s)$, then prove that

$$L\{F'''(t)\} = s^3 f(s) - s^2 F(0) - sF'(0) - F''(0) \text{ by assuming required conditions.}$$

44. If $F(t) = \begin{cases} t^2, & 0 < t \leq 1 \\ 0, & t > 1 \end{cases}$

(i) Find $L\{F''(t)\}$ (ii) Does the result

$$L\{F''(t)\} = s^2 L\{F(t)\} - sF(0) - F'(0) \text{ hold in this case? Explain.}$$

45. Find (i) $L\{t \cosh 3t\}$ (ii) $L\{t \sinh 2t\}$

46. Show that $L\left\{\int_0^t \frac{1 - e^{-u}}{u} du\right\} = \frac{1}{s} \log\left(1 + \frac{1}{s}\right)$

47. Given that $L\{J_0(t)\} = \frac{1}{\sqrt{1+s^2}}$, show that $\int_0^{\infty} te^{-3t} J_0(4t) dt = \frac{3}{125}$

48. Find $L\left\{\frac{e^{4t} \sin 3t}{t}\right\}$

49. Find $L\left\{\frac{1 - \cos at}{t}\right\}$

50. Find $L\left\{\int_0^t te^{-3t} \sin 4t dt\right\}$

UNIT II
INVERSE LAPLACE TRANSFORMS

I. Objective, Short answer questions of 2 marks each.

1. State Lerch's theorem for uniqueness of inverse Laplace Transform.
2. If c_1 and c_2 are any two constants, $f_1(s)$ and $f_2(s)$ are Laplace transforms of $F_1(t)$ and $F_2(t)$ respectively then prove that

$$L^{-1}\{c_1 f_1(s) + c_2 f_2(s)\} = c_1 L^{-1}\{f_1(s)\} + c_2 L^{-1}\{f_2(s)\}$$

3. Find $L^{-1}\left\{\frac{3}{s+4}\right\}$.
4. Find $L^{-1}\left\{\frac{1}{s^5}\right\}$.
5. Find $L^{-1}\left\{\frac{1}{2s-5}\right\}$.
6. Find $L^{-1}\left\{\frac{2s-5}{s^2-9}\right\}$.
7. Find $L^{-1}\left\{\frac{1}{s^{1/2}}\right\}$.
8. Find $L^{-1}\left\{\frac{12}{4-3s}\right\}$.
9. Find $L^{-1}\left\{\frac{8s}{s^2+16}\right\}$.
10. Find $L^{-1}\left\{\frac{s+1}{s^{4/3}}\right\}$.
11. If $L^{-1}\{f(s)\} = F(t)$, then prove that $L^{-1}\{f(s-k)\} = e^{kt}F(t)$.

12. Find $L^{-1}\left\{\frac{1}{\sqrt{2s+3}}\right\}$.

13. Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$.

14. If $L^{-1}\{f(s)\} = F(t)$, and $F(0) = 0$ prove that $L^{-1}\{sf(s)\} = F'(t)$.

15. Find $L^{-1}\left\{\frac{s}{s^2+1}\right\}$.

16. If $L^{-1}\{f(s)\} = \sin t$ then $L^{-1}\{f(s-2)\} = \dots\dots\dots$

17. If $L^{-1}\{f(s)\} = e^{2t}$ then $L^{-1}\{f(s-5)\} = \dots\dots\dots$

18. If $L^{-1}\{f(s)\} = \sin t$ then $L^{-1}\{sf(s)\} = \dots\dots\dots$

19. If $L^{-1}\{f(s)\} = \tan t$ then $L^{-1}\{s^2 f(s)\} = \dots\dots\dots$

II. Multiple choice questions of 1 mark each. Choose correct answer.

1. $L^{-1}\left\{\frac{1}{s}\right\}$ is

- (a) t (b) $\frac{1}{t}$ (c) 1 (d) $\frac{1}{2}$

2. $L^{-1}\left\{\frac{1}{s^2}\right\}$ is

- (a) t^2 (b) $\frac{t^2}{2!}$ (c) t (d) 1

3. $L^{-1}\left\{\frac{1}{s-k}\right\}$ is

- (a) k (b) e^{kt} (c) $\sin kt$ (d) t

4. $L^{-1}\left\{\frac{1}{s-5}\right\}$ is.....

(a) $\frac{1}{5}$ (b) 5 (c) e^t (d) e^{5t}

5. $L^{-1}\left\{\frac{1}{s^2 + k^2}\right\}$ is.....

(a) $\cos kt$ (b) $\frac{\sin kt}{k}$ (c) $\frac{\sin kt}{k^2}$ (d) $\cos kt$

6. $L^{-1}\left\{\frac{s}{s^2 + k^2}\right\}$ is.....

(a) $\cos kt$ (b) $\frac{\cos kt}{k}$ (c) $\frac{\sin kt}{k}$ (d) e^{kt}

7. $L^{-1}\left\{\frac{1}{s^2 - k^2}\right\}$ is.....

(a) $\cosh kt$ (b) $\sinh kt$ (c) $\frac{\sinh kt}{k}$ (d) $\frac{\sin kt}{k}$

8. $L^{-1}\left\{\frac{s}{s^2 - k^2}\right\}$ is.....

(a) $\cosh kt$ (b) $\frac{\cosh kt}{k}$ (c) $\sinh kt$ (d) $\frac{\sinh kt}{k}$

9. $L^{-1}\left\{\frac{1}{s^{n+1}}\right\}$ is.....

(a) $\frac{t^n}{(n+1)!}$ (b) $\frac{t^{n+1}}{(n)!}$ (c) $\frac{t^n}{\Gamma n}$ (d) $\frac{t^n}{\Gamma n+1}$

10. $L^{-1}\left\{\frac{1}{s^{5/2}}\right\}$ is.....

(a) $\frac{t^{3/2}}{\Gamma 5/2}$ (b) $\frac{t^{5/2}}{\Gamma 3/2}$ (c) $\frac{t^{3/2}}{\Gamma 3/2}$ (d) $\frac{t^{7/2}}{\Gamma 5/2}$

III. Questions of 3 marks each.

1. Find $L^{-1}\left\{\left(\frac{\sqrt{s}-1}{s}\right)^2\right\}$.

2. Find $L^{-1}\left\{\frac{2s+1}{s(s+1)}\right\}$.

3. Find $L^{-1}\left\{\frac{3s-8}{4s^2+25}\right\}$.

4. Find $L^{-1}\left\{\frac{5s+10}{9s^2-16}\right\}$

5. Find $L^{-1}\left\{\frac{3s-14}{s^2-4s+8}\right\}$

6. Find $L^{-1}\left\{\frac{8s+20}{s^2-12s+32}\right\}$

7. If $L^{-1}\{f(s)\} = F(t)$, then prove that $L^{-1}\{e^{-ks}f(s)\} = G(t) = \begin{cases} F(t-k) & , t > k \\ 0 & , t < k \end{cases}$

8. Find $L^{-1}\left\{\frac{e^{3s}}{(s-4)^2}\right\}$

9. Find $L^{-1}\left\{\frac{(s+1)e^{-\pi s}}{s^2+s+1}\right\}$

10. If $L^{-1}\{f(s)\} = F(t)$, and $F(0) = 0$ prove that $L^{-1}\{f(ks)\} = \frac{1}{k}F'\left(\frac{t}{k}\right)$

11. Find $L^{-1}\left\{\frac{1}{\sqrt{s^2-4s+20}}\right\}$

12. Find $L^{-1}\left\{\frac{e^{-s}}{\sqrt{s+1}}\right\}$

13. Find $L^{-1}\left\{\frac{se^{-2s}}{s^2 + 3s + 2}\right\}$.

14. If $L^{-1}\left\{\frac{s}{(s+1)^2}\right\} = \frac{1}{2}t \sin t$, find $L^{-1}\left\{\frac{32}{(16s^2 + 1)^2}\right\}$

15. If $L^{-1}\{f(s)\} = F(t)$, then prove that $L^{-1}\{f^n(s)\} = (-1)^n t^n F(t)$, $n = 1, 2, 3, \dots$

16. Find $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$.

17. Find $L^{-1}\left\{\frac{1}{(s-a)^3}\right\}$ given that $L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

18. Find $L^{-1}\left\{\frac{s}{(s^2 - a^2)^2}\right\}$ given that $L^{-1}\left\{\frac{1}{s^2 - a^2}\right\} = \frac{\sinh at}{a}$

19. Find $L^{-1}\left\{\frac{s+1}{(s^2 + 2s + 2)^2}\right\}$.

20. Find $L^{-1}\left\{\log\left(\frac{s+2}{s+1}\right)\right\}$.

21. Find $L^{-1}\left\{\log\left(1 + \frac{1}{s^2}\right)\right\}$

22. Find $L^{-1}\left\{\left(\frac{1}{s+a}\right)^3\right\}$

23. Find $L^{-1}\left\{\log\left(\frac{s+3}{s+2}\right)\right\}$

24. If $L^{-1}\{f(s)\} = F(t)$, then prove that $L^{-1}\left\{\int_0^\infty f(u)du\right\} = \frac{F(t)}{t}$

25. Find $L^{-1} \left\{ \int_s^{\infty} \left(\frac{1}{u} - \frac{1}{u+1} \right) du \right\}$.

26. If $L^{-1}\{f(s)\} = F(t)$, then prove that $L^{-1} \left\{ \frac{f(s)}{s} \right\} = \int_0^t F(u) du$.

27. prove that $L^{-1} \left\{ \frac{f(s)}{s^2} \right\} = \int_0^t \int_0^u F(v) dudv$

28. Find $L^{-1} \left\{ \frac{1}{s^3(s^2+1)} \right\}$

29. Find $L^{-1} \left\{ \frac{1}{s^3(s+1)} \right\}$

30. Find $L^{-1} \left\{ \frac{1}{s} \log \left(\frac{s+3}{s+2} \right) \right\}$.

31. Find $L^{-1} \left\{ \frac{1}{s^4(s^2+1)} \right\}$.

32. If $L^{-1}\{f(s)\} = F(t)$, then prove that $L^{-1}\{sf'(s)\} = -tF'(t) - F(t)$

33. If $L^{-1}\{f(s)\} = F(t)$, then prove that $L^{-1}\{sf''(s)\} = t^2F'(t) + 2tF(t)$

IV. Examples of 2 marks each

1. Find $L^{-1} \left\{ \frac{2s+1}{s(s+1)} \right\}$.

2. Find $L^{-1} \left\{ \frac{4s+12}{(s^2+8s+16)} \right\}$.

3. Find $L^{-1} \left\{ \frac{12}{4-3s} \right\}$

4. Find $L^{-1}\left\{\frac{8s}{(s^2 + 16)}\right\}$.

5. Find $L^{-1}\left\{\frac{3s + 7}{(s^2 - 2s - 3)}\right\}$.

6. Find $L^{-1}\left\{\frac{e^{-2s}}{(s^2)}\right\}$.

7. Find $L^{-1}\left\{\frac{8e^{-3s}}{(s^2 + 4)}\right\}$.

8. Find $L^{-1}\left\{\log\left(\frac{s + 3}{s + 2}\right)\right\}$.

9. Find $L^{-1}\left\{\frac{s}{s^2 + 1}\right\}$ by using the theorem . If $L^{-1}\{f(s)\} = F(t)$, $F(0) = 0$

then $L^{-1}\{sf(s)\} = F'(t)$

UNIT III
CONVOLUTION THEOREM

I. Objective type short questions of 2 marks each.

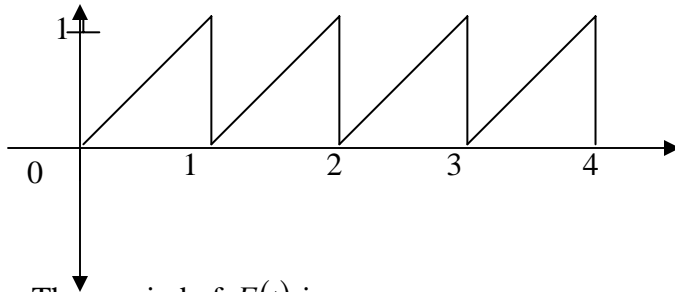
1. State convolution theorem.
2. Prove that $F * G = G * F$.
3. Prove that $F * (G * H) = (F * G) * H$.
4. Prove that $F * (G + H) = F * G + F * H$.
5. Using convolution theorem find $L^{-1}\left\{\frac{1}{(s+3)(s-1)}\right\}$.
6. Show that $\int_0^t \int_0^v F(u) du dv = \int_0^t (t-u)F(u) du$.
7. show that $1 * 1 * 1 * \dots * 1 = \frac{t^{n-1}}{(n-1)!}$, $n = 1, 2, 3, \dots$
8. If $F(t) = t^2$, $0 < t < 2$ and $F(t+2) = F(t)$, find $L\{F(t)\}$.
9. Find $L\{F(t)\}$ where $F(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ and $F(t+2) = F(t)$, $t > 0$.
10. Find $L^{-1}\left\{\frac{1}{(s+1)(s-2)}\right\}$.
11. Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$.
12. Find $L^{-1}\left\{\frac{1}{(s+2)^2(s-2)}\right\}$ using convolution theorem.

II. Multiple choice questions of 1 mark each.

1. If $F(t+2) = F(t)$ then $F(t)$ is periodic with period -----

- (a) π (b) 2π (c) 2 (d) 1

2. If $F(t)$ is periodic function shown graphically as follows.



Then period of $F(t)$ is -----

- (a) 1 (b) 2 (c) 3 (d) 4

3. $F * G =$ -----

- (a) $\int_0^{\infty} F(u)G(t-u)du$ (b) $\int_0^t F(u)G(t)du$ (c) $\int_0^t F(u)G(t-u)du$ (d) $\int_0^{\infty} F(u)G(t)du$

4. $1 * 1 * 1 =$ -----

- (a) $\frac{t^2}{2}$ (b) $\frac{t^3}{3!}$ (c) t^2 (d) $\frac{t}{2!}$

III. Questions of 4 marks each.

1. Let $F(t)$ have period $T > 0$ so that $F(t+T) = F(t)$. Prove that

$$L\{F(t)\} = \frac{\int_0^T e^{-st} F(t) dt}{1 - e^{-sT}}$$

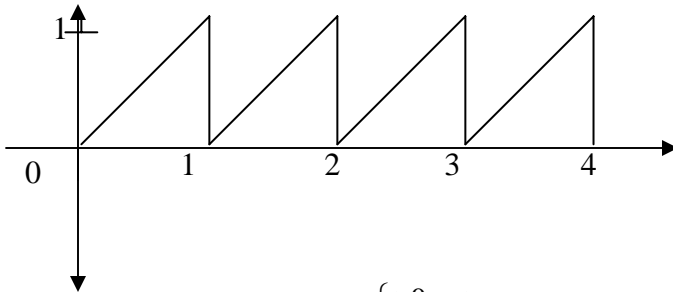
2. Graph the function $F(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$ and find $L\{F(t)\}$

3. Let $F(t) = \begin{cases} 3t, & 0 < t < 2 \\ 6, & 2 < t < 4 \end{cases}$ where $F(t)$ is periodic with period 4.

- (i) Graph $F(t)$

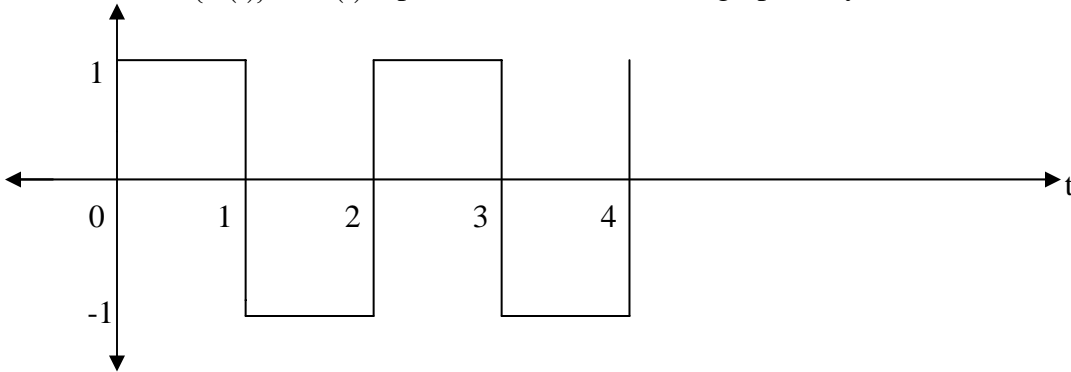
(ii) Find $L\{F(t)\}$

4. Find $L\{F(t)\}$ where $F(t)$ is periodic function shown graphically as below.



5. Find $L\{F(t)\}$ if $F(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$ and $F(t) = F(t + 2\pi)$

6. Find $L\{F(t)\}$ if $F(t)$ is periodic function shown graphically as below.



7. State and prove convolution theorem.

8. Using convolution theorem evaluate $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$.

9. Using convolution theorem evaluate $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$.

10. Using convolution theorem verify that $\int_0^t \sin u \cos(t-u) du = \frac{1}{2} t \sin t$

11. Using convolution theorem evaluate $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$.

12. Using convolution theorem evaluate $L^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$.

13. Using convolution theorem evaluate $L^{-1}\left\{\frac{1}{(s^2+1)^3}\right\}$.

14. Using convolution theorem evaluate $L^{-1}\left\{\frac{s}{(s^2+4)^3}\right\}$.

15. Using convolution theorem evaluate $L^{-1}\left\{\frac{2s-1}{s^3-s}\right\}$.

16. Find $L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$.

17. Find $L^{-1}\left\{\frac{s+1}{6s^2+7s+2}\right\}$.

18. Find $L^{-1}\left\{\frac{s^2-15s-11}{(s+1)(s-1)^2}\right\}$.

19. Find $L^{-1}\left\{\frac{5s^2-15s-11}{(s+1)(s-2)^3}\right\}$.

20. Find $L^{-1}\left\{\frac{27-12s}{(s+4)(s^2+9)}\right\}$.

21. Find $L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}$.

22. Find $L^{-1}\left\{\frac{s-1}{(s+3)(s^2+2s+2)}\right\}$.

23. Find $L^{-1}\left\{\frac{3s+16}{(s^2-s-6)}\right\}$.

24. Find $L^{-1}\left\{\frac{2s+1}{(s-1)(s-2)(s+3)}\right\}$.

25. Find $L^{-1}\left\{\frac{1}{(s-1)^2(s+2)}\right\}$.

26. Find $L^{-1}\left\{\frac{s^2}{(s+2)^3}\right\}$.

27. Find $L^{-1}\left\{\frac{s^3-3}{(s+2)(s-3)(s^2+2s+5)}\right\}$.

28. Find $L^{-1}\left\{\frac{s}{(s^2-2s+2)(s^2+2s+2)}\right\}$.

UNIT IV
APPLICATIONS TO DIFFERENTIAL EQUATIONS

I. Objective type short questions of 2 marks each.

1. Define Heavisides unit step function $U(t-a)$.
2. Prove that $L\{U(t-a)\} = \frac{e^{-as}}{s}$ where $U(t-a)$ is Heavisides unit step.
3. Define Dirac-delta function $\delta(t-a)$.
4. Prove that $L\{\delta(t-a)\} = e^{-as}$.
5. Find $L\{\sin 2t\delta(t-2)\}$.
6. Find $L\{t\delta(t-3)\}$.

7. Find $L\{\cos 2t\delta(t-3)\}$.

8. Find $L\{t^2U(t-2)\}$.

9. Find $L\{t^2\delta(t-4)\}$.

10. Find $L\{t^3\delta(t-2)\}$.

11. Find $L\left\{\sin 2t\delta\left(t-\frac{\pi}{4}\right)\right\}$.

12. Find $L\left\{\cos 2t\delta\left(t-\frac{\pi}{4}\right)\right\}$.

II. Multiple choice questions of 1 mark each.

5. If $U(t-a)$ is Heavisides unit step function then $L\{U(t-a)\}$ is -----

(a) e^{-as} (b) se^{-as} (c) $\frac{1}{s}e^{-as}$ (d) $\frac{s}{e^{-as}}$

6. $L\{U(t-4)\}$ is -----

(a) e^{-4s} (b) se^{-4s} (c) $\frac{1}{se^{4s}}$ (d) $\frac{s}{e^{-4s}}$

7. $L\{U(t-2)\}$ is -----

(a) $\frac{e^{-2s}}{s}$ (b) se^{-2s} (c) se^{2s} (d) $\frac{s}{e^{-2s}}$

8. $L\{F(t-2)U(t-2)\}$ is -----

(a) $\frac{e^{-2s}}{f(s)}$ (b) $f(s)e^{-2s}$ (c) $f(s)e^{2s}$ (d) $\frac{1}{f(s)e^{-2s}}$

9. $L\{\delta(t-2)\}$ is -----

(a) e^{-as} (b) e^{-as} (c) se^{-as} (d) $\frac{s}{e^{-as}}$

10. $L\{\delta(t-5)\}$ is -----

(a) e^{5s} (b) $e^{5/s}$ (c) $\frac{1}{e^{5s}}$ (d) $\frac{s}{e^{5s}}$

11. $L\{\delta(t)\}$ is -----

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{2}{3}$

12. $L\{F(t)\delta(t-4)\}$ is -----

(a) $F(4)e^{-4s}$ (b) $F(2)e^{-4s}$ (c) $\frac{F(4)}{e^{-4s}}$ (d) $\frac{e^{-4s}}{F(2)}$

III. Questions of 6 marks each.

1. (a) Explain the method of solving the linear differential equation

$$\frac{d^2Y}{dt^2} + A\frac{dY}{dt} + BY = F(t)$$

subject to $Y(0) = \alpha$, $Y'(0) = \beta$ by using Laplace transform.

(b) Solve the differential equation $Y'' + 4Y' + 4Y = 4e^{-2t}$ subject to condition

$Y(0) = -1$, $Y'(0) = -4$ by using Laplace transform.

2. (a) Explain the method of solving the linear differential equation

$$\frac{d^2Y}{dt^2} + A\frac{dY}{dt} + BY = F(t) \text{ where } A, B \text{ are constants}$$

subject to initial or boundary conditions $Y(0) = \alpha$, $Y'(0) = \beta$, α, β are given constants.

(b) Solve the differential equation $Y'' + Y = t$ subject to condition

$Y(0) = 1$, $Y'(0) = -2$ by using Laplace transform.

3. (a) Explain the method of solving the linear differential equation

$$\frac{d^2Y}{dt^2} + A \frac{dY}{dt} + BY = F(t) \text{ where } A, B \text{ are constants}$$

subject to initial or boundary conditions $Y(0) = \alpha$, $Y'(0) = \beta$, α , β are given constants.

(b) Solve the differential equation $Y'' - 2Y' + 2Y = 0$ subject to condition $Y(0) = Y'(0) = 1$ by using Laplace transform

4. Solve the differential equation $Y'' + 9Y = \cos 2t$ if $Y(0) = 1$, $Y\left(\frac{\pi}{2}\right) = -1$.

5. Solve the differential equation $Y''' - Y' = 2 \cos t$ if $Y(0) = 3$, $Y'(0) = 2$, $Y''(0) = 1$.

6. Solve the differential equation $(D^3 + 1)Y = 1$, $t > 0$ if $Y = DY = D^2Y = 0$ when $t = 0$.

7. (a) Solve $Y'' + k^2Y = F(t)$, subject to $Y(0) = 1$, $Y'(0) = -2$.

(b) Find the general solution of $Y'' - a^2Y = F(t)$.

8. (a) Explain the method of solving the linear differential equation

$$\frac{dY}{dt} + a_1Y + a_2 \int_0^t Y(t) dt = F(t), t > 0$$

(b) Solve the differential equation $Y'' + 4Y = 9t$, $Y(0) = 0$, $Y'(0) = 7$ subject to condition $Y(0) = Y'(0) = 1$ by using Laplace transform

9. Solve the differential equations

(a) $Y''(t) - 3Y'(t) + 2Y(t) = 4t + 12e^{-t}$, subject to $Y(0) = 6$, $Y'(0) = -1$.

(b) $Y''(t) - 4Y'(t) + 5Y(t) = 125t^2$, subject to $Y(0) = Y'(0) = 0$..

10. Solve the differential equations

(a) $Y''(t) + Y(t) = 8 \cos t$, subject to $Y(0) = 1$, $Y'(0) = -1$.

(b) $Y'''(t) - Y(t) = e^t$, subject to $Y(0) = Y'(0) = Y''(0) = 0$.

11. Solve the differential equations

(a) $Y''(t) + 9Y(t) = 18t$, subject to $Y(0) = 0$, $Y'\left(\frac{\pi}{2}\right) = 0$.

(b) $Y''(t) - 4Y'(t) + 3Y(t) = F(t)$, subject to $Y(0) = 1$, $Y'(0) = 0$.

12. Solve the differential equations

(a) $(D^2 - D - 2)Y = 20 \sin 2t$, $Y(0) = -1$, $Y'(0) = 2$.

(b) $(D^2 + 2D + 1)Y = t$, $Y(0) = -3$, $Y(1) = -1$.

13. Solve $(D^3 - D^2 - D + 1)Y = 8te^{-t}$ if $Y(0) = Y''(0) = 0$, $Y'(0) = 1$

14. Solve $\frac{dY}{dt} + 3Y + 2 \int_0^t Y(t) dt = t$, subject to $Y(0) = 0$.

15. Solve $Y'(t) + 5 \int_0^t \cos 2(t-u)Y(u) du = 10$, if $Y(0) = 2$.

16. Solve the following equation for $Y(t)$ with the condition $Y(0) = 0$

$$Y'(t) = \sin t + \int_0^t Y(t-u) \cos u du.$$

17. Solve $X'(t) + \int_0^t X(u) \cos(t-u) du = 0$, if $X(0) = 1$.

18. Find (a) $L\{t^4 U(t-2)\}$ (b) $L\{t^2 U(t-4)\}$.

19. $F(t) = \begin{cases} t^2, & 0 < t < 1 \\ 4t, & t > 1 \end{cases}$

Find $L\{F(t)\}$ by expressing it in terms of Heavisides unit step function.

20. (a) Prove that $L\{F(t)\delta(t-a)\} = F(a)e^{-as}$

(b) Find $L\{tU(t-4) - t^3\delta(t-2)\}$

21. Find (a) $L\{t^3U(t-4) - t^2\delta(t-4)\}$

(b) $L\{tU(t-4) + t^2\delta(t-4)\}$

IV. Examples of 2 marks each.

1. Solve the differential equation $\frac{d^2Y}{dt^2} + Y = 0$, subject to conditions

$Y(0) = 1, \left(\frac{dY}{dt}\right)_{t=0} = 0$ by using Laplace transform.

2. Solve $Y'' - 2Y' + 2Y = 0$ subject to $Y(0) = Y'(0) = 1$.

3. Evaluate $\int_0^{\infty} e^{-4t} \delta(t-2) dt$.

4. Find $L\{(1 + 2t - 3t^2 + 4t^3)U(t-2)\}$

5. Find $L\{(1 + 2t - t^2 + t^3)U(t-1)\}$.

6. Find $L\{tU(t-4)\}$.

7. Find $L\{t^2U(t-4)\}$.